1a $\sin\alpha = \dfrac{\text{overstaande rechthoekszijde}}{\text{schuine zijde}} \Rightarrow$ (in figuur 6.1) $\sin 65^\circ = \dfrac{PQ}{OP} = \dfrac{PQ}{1} \Rightarrow PQ = 1\cdot\sin 65^\circ \approx 0{,}91.$

$\cos\alpha = \dfrac{\text{aanliggende rechthoekszijde}}{\text{schuine zijde}} \Rightarrow$ (in figuur 6.1) $\cos 65^\circ = \dfrac{OQ}{OP} = \dfrac{OQ}{1} \Rightarrow OQ = 1\cdot\cos 65^\circ \approx 0{,}42.$

1b $P(0{,}42;\ 0{,}91).$

1c $\angle POQ = 180^\circ - 115^\circ = 65^\circ;\ PQ \approx 0{,}91$ en $OQ \approx 0{,}42 \Rightarrow P(-0{,}42;\ 0{,}91).$
(P in figuur 6.2 is het spiegelbeeld van P in figuur 6.1 ten opzichte van de y-as)

1d $\cos 115^\circ \approx -0{,}42$ en $\sin 115^\circ \approx 0{,}91$. Dus $\cos 115^\circ = x_P$ en $\sin 115^\circ = y_P$.

2a $\alpha = 0^\circ \Rightarrow P = (1,\ 0) \Rightarrow \sin 0^\circ = y_P = 0.$

2b $\alpha = 0^\circ \Rightarrow P = (1,\ 0) \Rightarrow \cos 0^\circ = x_P = 1.$

2c $\alpha = 90^\circ \Rightarrow P = (0,\ 1) \Rightarrow \sin 90^\circ = y_P = 1.$

2d $\alpha = 90^\circ \Rightarrow P = (0,\ 1) \Rightarrow \cos 90^\circ = x_P = 0.$

2e $\alpha = 270^\circ \Rightarrow P = (0,\ -1) \Rightarrow \sin 270^\circ = y_P = -1.$

2f $\alpha = 270^\circ \Rightarrow P = (0,\ -1) \Rightarrow \cos 270^\circ = x_P = 0.$

2g $\alpha = 360^\circ \Rightarrow P = (1,\ 0) \Rightarrow \sin 360^\circ = y_P = 0.$

2h $\alpha = 360^\circ \Rightarrow P = (1,\ 0) \Rightarrow \cos 360^\circ = x_P = 1.$

2i $\alpha = 450^\circ \Rightarrow P = (0,\ 1) \Rightarrow \sin 450^\circ = y_P = 1.$

2j $\alpha = -90^\circ \Rightarrow P = (0,\ -1) \Rightarrow \cos(-90^\circ) = x_P = 0.$

2k $\alpha = -540^\circ \Rightarrow P = (-1,\ 0) \Rightarrow \sin(-540^\circ) = y_P = 0.$

2l $\alpha = -180^\circ \Rightarrow P = (-1,\ 0) \Rightarrow \cos(-180^\circ) = x_P = -1.$

3a $\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}.$

3b $\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}.$

3c $\sin(-135^\circ) = \sin(-180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{1}{2}\sqrt{2}.$

3d $\cos(-135^\circ) = \cos(-180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}.$

3e $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{1}{2}\sqrt{3}.$

3f $\cos 300^\circ = \cos(360^\circ - 60^\circ) = \cos 60^\circ = \frac{1}{2}.$

4a Zie het scherm hiernaast.

4b $x_P = \cos 110^\circ \approx -0{,}34$ en $y_P = \sin 110^\circ \approx 0{,}94.$
$x_Q = \cos 200^\circ \approx -0{,}94$ en $y_Q = \sin 200^\circ \approx -0{,}34.$
$x_R = \cos(-102^\circ) \approx -0{,}21$ en $y_R = \sin(-102^\circ) \approx -0{,}98.$
$x_S = \cos(-50^\circ) \approx 0{,}64$ en $y_S = \sin(-50^\circ) \approx -0{,}77.$

5 De cirkel is een vergroting van de eenheidscirkel met factor 2.
$x_B = 2\cdot\cos 72^\circ \approx 0{,}62$ en $y_B = 2\cdot\sin 72^\circ \approx 1{,}90.$
$x_C = 2\cdot\cos 144^\circ \approx -1{,}62$ en $y_C = 2\cdot\sin 144^\circ \approx 1{,}18.$
$x_D = 2\cdot\cos 216^\circ \approx -1{,}62$ en $y_D = 2\cdot\sin 216^\circ \approx -1{,}18.$
$x_E = 2\cdot\cos 288^\circ \approx 0{,}62$ en $y_E = 2\cdot\sin 288^\circ \approx -1{,}90.$

6a omtrek $= 2\pi r = 2\pi\cdot 1 = 2\pi.$

6b $\alpha = 90^\circ \Rightarrow$ een kwart van de eenheidscirkel doorlopen $\Rightarrow$ lengte van de doorlopen boog is $\frac{1}{4}\cdot 2\pi = \frac{2}{4}\pi = \frac{1}{2}\pi.$

6c $\alpha = 180^\circ \Rightarrow$ een halve eenheidscirkel doorlopen $\Rightarrow$ lengte van de doorlopen boog is $\frac{1}{2}\cdot 2\pi = \pi.$

6d $1\frac{1}{2}\pi = \frac{3}{4}\cdot 2\pi \Rightarrow$ driekwart van de omtrek van de eenheidscirkel $\Rightarrow \alpha = \frac{3}{4}\cdot 360^\circ = 3\cdot 90^\circ = 270^\circ.$

π radialen $= 180^\circ \Rightarrow 1$ radiaal $= \dfrac{180^\circ}{\pi}$ en $\dfrac{1}{180}\pi$ radialen $= 1^\circ.$

7a $\frac{1}{6}\pi$ rad $= \frac{1}{6}\cdot 180^\circ = 30^\circ.$

7b $\frac{1}{4}\pi$ rad $= \frac{1}{4}\cdot 180^\circ = 45^\circ.$

7c 2π rad $= 2\cdot 180^\circ = 360^\circ.$

7d 2 rad $= 2\cdot\frac{180^\circ}{\pi} \approx 114{,}6^\circ.$

7e $\frac{5}{4}\pi$ rad $= \frac{5}{4}\cdot 180^\circ = 225^\circ.$

7f $\frac{5}{4}$ rad $= \frac{5}{4}\cdot\frac{180^\circ}{\pi} \approx 71{,}6^\circ.$

7g $-2\frac{1}{3}\pi$ rad $= -2\frac{1}{3}\cdot 180^\circ = -420^\circ.$

7h $-2\frac{1}{3}$ rad $= -2\frac{1}{3}\cdot\frac{180^\circ}{\pi} \approx -133{,}7^\circ.$

8a $360^\circ = \frac{360}{180}\cdot\pi$ rad $= 2\pi$ rad.

8b $30^\circ = \frac{30}{180}\cdot\pi$ rad $= \frac{1}{6}\pi$ rad.

8c $45^\circ = \frac{45}{180}\cdot\pi$ rad $= \frac{1}{4}\pi$ rad.

8d $60^\circ = \frac{60}{180}\cdot\pi$ rad $= \frac{1}{3}\pi$ rad.

8e $90^\circ = \frac{90}{180}\cdot\pi$ rad $= \frac{1}{2}\pi$ rad.

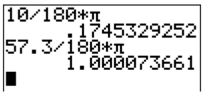
8f $135^\circ = \frac{135}{180}\cdot\pi$ rad $= \frac{3}{4}\pi$ rad.

8g $300^\circ = \frac{300}{180}\cdot\pi$ rad $= 1\frac{2}{3}\pi$ rad.

8h $210^\circ = \frac{210}{180}\cdot\pi$ rad $= 1\frac{1}{6}\pi$ rad.

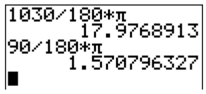
9a $10^\circ = \frac{10}{180} \cdot \pi$ rad $\approx 0{,}17$ rad.

9b $57{,}3^\circ = \frac{57{,}3}{180} \cdot \pi$ rad $\approx 1{,}00$ rad.



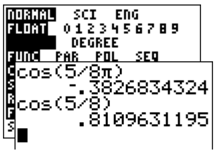
9c $1030^\circ = \frac{1030}{180} \cdot \pi$ rad $\approx 17{,}98$ rad.

9d $90^\circ = \frac{90}{180} \cdot \pi$ rad $\approx 1{,}57$ rad.



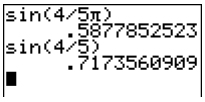
10a $\cos\left(\frac{5}{8}\pi\right) \approx -0{,}38.$

10b $\cos\left(\frac{5}{8}\right) \approx 0{,}81.$



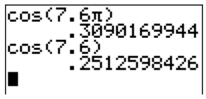
10c $\sin\left(\frac{4}{5}\pi\right) \approx 0{,}59.$

10d $\sin\left(\frac{4}{5}\right) \approx 0{,}72.$



10e $\cos(7{,}6\pi) \approx 0{,}31.$

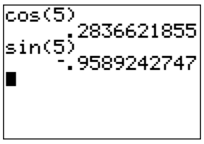
10f $\cos(7{,}6) \approx 0{,}25.$

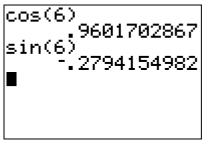


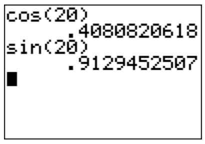
11a $x_P = \cos(5) \approx 0{,}28$ en $y_P = \sin(5) \approx -0{,}96.$

11b $x_P = \cos(6) \approx 0{,}96$ en $y_P = \sin(6) \approx -0{,}28.$

11c $x_P = \cos(20) \approx 0{,}41$ en $y_P = \sin(20) \approx 0{,}92.$







12a $\cos\left(\frac{1}{6}\pi\right) = \cos 30^\circ = \frac{1}{2}\sqrt{3}.$

12b $\sin\left(\frac{1}{4}\pi\right) = \sin 45^\circ = \frac{1}{2}\sqrt{2}.$

hoek	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
sinus	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cosinus	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

Leer deze tabel uit het hoofd.

13a $\sin\left(\frac{3}{4}\pi\right) = \sin\left(\pi - \frac{1}{4}\pi\right) = \sin\left(\frac{1}{4}\pi\right) = \frac{1}{2}\sqrt{2}.$

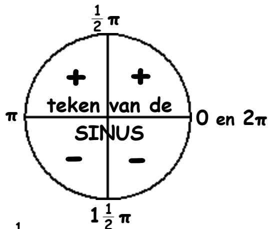
13b $\cos\left(\frac{7}{6}\pi\right) = \cos\left(\pi + \frac{1}{6}\pi\right) = -\cos\left(\frac{1}{6}\pi\right) = -\frac{1}{2}\sqrt{3}.$

13c $\sin\left(1\frac{1}{3}\pi\right) = \sin\left(\pi + \frac{1}{3}\pi\right) = -\sin\left(\frac{1}{3}\pi\right) = -\frac{1}{2}\sqrt{3}.$

13d $\cos\left(\frac{5}{3}\pi\right) = \cos\left(-\frac{1}{3}\pi\right) = \cos\left(\frac{1}{3}\pi\right) = \frac{1}{2}.$

13e $\cos\left(1\frac{1}{3}\pi\right) = \cos\left(\pi + \frac{1}{3}\pi\right) = -\cos\left(\frac{1}{3}\pi\right) = -\frac{1}{2}.$

13f $\sin\left(-\frac{1}{4}\pi\right) = -\sin\left(\frac{1}{4}\pi\right) = -\frac{1}{2}\sqrt{2}.$



14a $\sin(\alpha) = \frac{1}{2}\sqrt{3} = \sin\left(\frac{1}{3}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{3}\pi$ of $\alpha = \pi - \frac{1}{3}\pi = \frac{2}{3}\pi.$

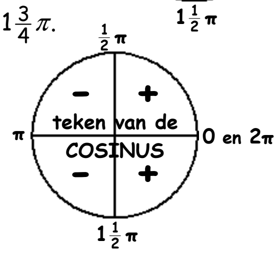
14b $\cos(\alpha) = -\frac{1}{2} = -\cos\left(\frac{1}{3}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi - \frac{1}{3}\pi = \frac{2}{3}\pi$ of $\alpha = \pi + \frac{1}{3}\pi = 1\frac{1}{3}\pi.$

14c $\sin(\alpha) = -\frac{1}{2}\sqrt{2} = -\sin\left(\frac{1}{4}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi + \frac{1}{4}\pi = 1\frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$

14d $\cos(\alpha) = 0 = \cos\left(\frac{1}{2}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{2}\pi$ of $\alpha = 2\pi - \frac{1}{2}\pi = 1\frac{1}{2}\pi.$

14e $\cos(\alpha) = \frac{1}{2}\sqrt{3} = \cos\left(\frac{1}{6}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = 2\pi - \frac{1}{6}\pi = 1\frac{5}{6}\pi.$

14f $\cos(\alpha) = \frac{1}{2}\sqrt{2} = \cos\left(\frac{1}{4}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$



15 Bij de snijpunten met de y-as horen de draaiïngshoeken ..., $-2\frac{1}{2}\pi$, $-1\frac{1}{2}\pi$, $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$, $1\frac{1}{2}\pi$, $-2\frac{1}{2}\pi$, ...

16a $\sin(3x - \frac{1}{2}\pi) = 0 = \sin(0)$
$3x - \frac{1}{2}\pi = 0 + k \cdot \pi$
$3x = \frac{1}{2}\pi + k \cdot \pi$
$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi.$

16b $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0 = \cos(\frac{1}{2}\pi)$
$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$
$x = 1\frac{1}{3}\pi + k \cdot 2\pi.$

16c $\sin^2(x) - \sin(x) = 0$
$\sin(x) \cdot (\sin(x) - 1) = 0$
$\sin(x) = 0$ of $\sin(x) = 1$
$x = k \cdot \pi$ of $x = \frac{1}{2}\pi + k \cdot 2\pi.$

16d $\cos^2(2x) + \cos(2x) = 0$
$\cos(2x) \cdot (\cos(2x) + 1) = 0$
$\cos(2x) = 0$ of $\cos(2x) = -1$
$2x = \frac{1}{2}\pi + k \cdot \pi$ of $2x = \pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$ of $x = \frac{1}{2}\pi + k \cdot \pi.$

17a $\sin^2(2x) = 1$
$\sin(2x) = 1$ of $\sin(2x) = -1$
$2x = \frac{1}{2}\pi + k \cdot 2\pi$ of $2x = -\frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \pi$ of $x = -\frac{1}{4}\pi + k \cdot \pi.$

of sneller: 17abc $\sin^2(2x) = 1$
$\sin(2x) = \pm 1$
$2x = \frac{1}{2}\pi + k \cdot \pi$
$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

17b ..., $-2\frac{1}{4}\pi$, $-1\frac{1}{4}\pi$, $-\frac{1}{4}\pi$, $\frac{3}{4}\pi$, $1\frac{3}{4}\pi$, $2\frac{3}{4}\pi$, ...

17c ..., $-2\frac{1}{4}\pi$, $-1\frac{3}{4}\pi$, $-1\frac{1}{4}\pi$, $-\frac{3}{4}\pi$, $-\frac{1}{4}\pi$, $\frac{1}{4}\pi$, $\frac{3}{4}\pi$, $1\frac{1}{4}\pi$, $1\frac{3}{4}\pi$, $2\frac{1}{4}\pi$, $2\frac{3}{4}\pi$, ... is te schrijven als $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

18a $\cos^2(x-\frac{1}{5}\pi)=1$
$\cos(x-\frac{1}{5}\pi)=\pm1$
$x-\frac{1}{5}\pi=0+k\cdot\pi$
$x=\frac{1}{5}\pi+k\cdot\pi.$

18b $\sin^2(2x-\frac{1}{4}\pi)=1$
$\sin(2x-\frac{1}{4}\pi)=\pm1$
$2x-\frac{1}{4}\pi=\frac{1}{2}\pi+k\cdot\pi$
$2x=\frac{3}{4}\pi+k\cdot\pi$
$x=\frac{3}{8}\pi+k\cdot\frac{1}{2}\pi.$

18c $\sin^3(x)-\sin(x)=0$
$\sin(x)\cdot(\sin^2(x)-1)=0$
$\sin(x)=0$ of $\sin^2(x)=1$
$\sin(x)=0$ of $\sin(x)=\pm1$
$x=0+k\cdot\pi$ of $x=\frac{1}{2}\pi+k\cdot\pi$
$x=k\cdot\frac{1}{2}\pi.$

18d $\cos^3(2x)-\cos(2x)=0$
$\cos(2x)\cdot(\cos^2(x)-1)=0$
$\cos(2x)=0$ of $\cos^2(2x)=1$
$\cos(2x)=0$ of $\cos(2x)=\pm1$
$2x=\frac{1}{2}\pi+k\cdot\pi$ of $2x=0+k\cdot\pi$
$x=\frac{1}{4}\pi+k\cdot\frac{1}{2}\pi$ of $x=k\cdot\frac{1}{2}\pi$
$x=k\cdot\frac{1}{4}\pi.$

19a $\sin(4x-\frac{1}{3}\pi)=1$
$4x-\frac{1}{3}\pi=\frac{1}{2}\pi+k\cdot2\pi$
$4x=\frac{5}{6}\pi+k\cdot2\pi$
$x=\frac{5}{24}\pi+k\cdot\frac{1}{2}\pi.$

19b $\cos(4\pi x)=-1$
$4\pi x=\pi+k\cdot2\pi$
$x=\frac{1}{4}+k\cdot\frac{1}{2}.$

19c $\sin^2(\frac{1}{4}\pi x)=1$
$\sin(\frac{1}{4}\pi x)=\pm1$
$\frac{1}{4}\pi x=\frac{1}{2}\pi+k\cdot\pi$
$x=2+k\cdot4.$

19d $\sin(2x)\cdot\cos(2x)+\sin(2x)=0$
$\sin(2x)\cdot(\cos(2x)+1)=0$
$\sin(2x)=0$ of $\cos(2x)=-1$
$2x=0+k\cdot\pi$ of $2x=\pi+k\cdot2\pi$
$x=k\cdot\frac{1}{2}\pi$ of $x=\frac{1}{2}\pi+k\cdot\pi$
$x=k\cdot\frac{1}{2}\pi.$

20a $\sin(\frac{1}{6}\pi)=\frac{1}{2}$, dus $x=\frac{1}{6}\pi$ is een oplossing van $\sin(x)=\frac{1}{2}$.

20b $2\frac{1}{6}\pi=\frac{1}{6}\pi+2\pi\Rightarrow\sin(2\frac{1}{6}\pi)=\sin(\frac{1}{6}\pi)=\frac{1}{2}$ en $4\frac{1}{6}\pi=\frac{1}{6}\pi+2\cdot2\pi\Rightarrow\sin(4\frac{1}{6}\pi)=\sin(\frac{1}{6}\pi)=\frac{1}{2}$.

20c $\sin(\frac{5}{6}\pi)=\sin(\pi-\frac{1}{6}\pi)=\sin(\frac{1}{6}\pi)=\frac{1}{2}$, dus $x=\frac{5}{6}\pi$ is een oplossing van $\sin(x)=\frac{1}{2}$.

20d $2\frac{5}{6}\pi=\frac{5}{6}\pi+2\pi\Rightarrow\sin(2\frac{5}{6}\pi)=\sin(\frac{5}{6}\pi)=\sin(\frac{1}{6}\pi)=\frac{1}{2}$ en $-1\frac{1}{6}\pi=\frac{5}{6}\pi-2\pi\Rightarrow\sin(-1\frac{1}{6}\pi)=\sin(\frac{5}{6}\pi)=\sin(\frac{1}{6}\pi)=\frac{1}{2}$.

21a $2\sin(\frac{1}{2}x)=1$
$\sin(\frac{1}{2}x)=\frac{1}{2}$
$\frac{1}{2}x=\frac{1}{6}\pi+k\cdot2\pi$ of $\frac{1}{2}x=\frac{5}{6}\pi+k\cdot2\pi$
$x=\frac{1}{3}\pi+k\cdot4\pi$ of $x=\frac{5}{3}\pi+k\cdot4\pi.$

21b $2\cos(x-\frac{1}{3}\pi)=1$
$\cos(x-\frac{1}{3}\pi)=\frac{1}{2}$
$x-\frac{1}{3}\pi=\frac{1}{3}\pi+k\cdot2\pi$ of $x-\frac{1}{3}\pi=-\frac{1}{3}\pi+k\cdot2\pi$
$x=\frac{2}{3}\pi+k\cdot2\pi$ of $x=k\cdot2\pi.$

21c $2\sin(2x-\frac{1}{4}\pi)=-\sqrt{3}$
$\sin(2x-\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{3}$
$2x-\frac{1}{4}\pi=-\frac{1}{3}\pi+k\cdot2\pi$ of $2x-\frac{1}{4}\pi=1\frac{1}{3}\pi+k\cdot2\pi$
$2x=-\frac{1}{12}\pi+k\cdot2\pi$ of $2x=\frac{19}{12}\pi+k\cdot2\pi$
$x=-\frac{1}{24}\pi+k\cdot\pi$ of $x=\frac{19}{24}\pi+k\cdot\pi.$

21d $2\cos(3x-\pi)=-1$
$\cos(3x-\pi)=-\frac{1}{2}$
$3x-\pi=\frac{2}{3}\pi+k\cdot2\pi$ of $3x-\pi=-\frac{2}{3}\pi+k\cdot2\pi$
$3x=\frac{5}{3}\pi+k\cdot2\pi$ of $3x=\frac{1}{3}\pi+k\cdot2\pi$
$x=\frac{5}{9}\pi+k\cdot\frac{2}{3}\pi$ of $x=\frac{1}{9}\pi+k\cdot\frac{2}{3}\pi.$

22a $2\sin(2x-\frac{1}{6}\pi)=\sqrt{2}$
$\sin(2x-\frac{1}{6}\pi)=\frac{1}{2}\sqrt{2}$
$2x-\frac{1}{6}\pi=\frac{1}{4}\pi+k\cdot2\pi$ of $2x-\frac{1}{6}\pi=\frac{3}{4}\pi+k\cdot2\pi$
$2x=\frac{5}{12}\pi+k\cdot2\pi$ of $2x=\frac{11}{12}\pi+k\cdot2\pi$
$x=\frac{5}{24}\pi+k\cdot\pi$ of $x=\frac{11}{24}\pi+k\cdot\pi.$
x op $[0,2\pi]$ geeft
$x=\frac{5}{24}\pi$ of $x=1\frac{5}{24}\pi$ of $x=\frac{11}{24}\pi$ of $x=1\frac{11}{24}\pi.$

22b $2\cos(3x-\frac{1}{2}\pi)=\sqrt{3}$
$\cos(3x-\frac{1}{2}\pi)=\frac{1}{2}\sqrt{3}$
$3x-\frac{1}{2}\pi=\frac{1}{6}\pi+k\cdot2\pi$ of $3x-\frac{1}{2}\pi=-\frac{1}{6}\pi+k\cdot2\pi$
$3x=\frac{2}{3}\pi+k\cdot2\pi$ of $3x=\frac{1}{3}\pi+k\cdot2\pi$
$x=\frac{2}{9}\pi+k\cdot\frac{2}{3}\pi$ of $x=\frac{1}{9}\pi+k\cdot\frac{2}{3}\pi.$
x op $[0,2\pi]$ geeft
$x=\frac{2}{9}\pi$ of $x=\frac{8}{9}\pi$ of $x=\frac{14}{9}\pi$ of $x=\frac{1}{9}\pi$ of $x=\frac{7}{9}\pi$ of $x=\frac{13}{9}\pi.$

22c $\sin(\frac{2}{3}x)=-\frac{1}{2}\sqrt{2}$
$\frac{2}{3}x=-\frac{1}{4}\pi+k\cdot2\pi$ of $\frac{2}{3}x=\frac{5}{4}\pi+k\cdot2\pi$
$x=-\frac{3}{8}\pi+k\cdot3\pi$ of $x=\frac{15}{8}\pi+k\cdot3\pi.$
x op $[0,2\pi]$ geeft $x=\frac{15}{8}\pi.$

22d $\cos(\frac{1}{2}x)=-\frac{1}{2}\sqrt{3}$
$\frac{1}{2}x=\frac{5}{6}\pi+k\cdot2\pi$ of $\frac{1}{2}x=-\frac{5}{6}\pi+k\cdot2\pi$
$x=\frac{5}{3}\pi+k\cdot4\pi$ of $x=-\frac{5}{3}\pi+k\cdot4\pi.$
x op $[0,2\pi]$ geeft $x=\frac{5}{3}\pi.$

23a $2\sin^2(x)=1$
$\sin^2(x)=\frac{1}{2}$
$\sin(x)=\sqrt{\frac{1}{2}}=\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{4}}\cdot\sqrt{2}=\frac{1}{2}\sqrt{2}$ of $\sin(x)=-\sqrt{\frac{1}{2}}=-\frac{1}{2}\sqrt{2}.$

23b $\sin(x)=\frac{1}{2}\sqrt{2}$ geeft $x=\frac{1}{4}\pi+k\cdot 2\pi$ of $x=\frac{3}{4}\pi+k\cdot 2\pi.$
$\sin(x)-\frac{1}{2}\sqrt{2}$ geeft $x=-\frac{1}{4}\pi+k\cdot 2\pi$ of $x=\frac{5}{4}\pi+k\cdot 2\pi.$

of sneller: 23d $2\sin^2(x)=1$
$\sin^2(x)=\frac{1}{2}$
$\sin(x)=\pm\sqrt{\frac{1}{2}}=\pm\sqrt{\frac{2}{4}}=\pm\sqrt{\frac{1}{4}}\cdot\sqrt{2}=\pm\frac{1}{2}\sqrt{2}$
Uit de exacte-waarden-cirkel lees je af:
$x=\frac{1}{4}\pi+k\cdot\frac{1}{2}\pi.$

23c Uitschrijven geeft: ..., $-\frac{5}{4}\pi,\ -\frac{3}{4}\pi,\ -\frac{1}{4}\pi,\ \frac{1}{4}\pi,\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{7}{4}\pi,\ \frac{9}{4}\pi,\ \frac{11}{4}\pi,\ \frac{13}{4}\pi,\ \frac{15}{4}\pi,\ \ldots\Rightarrow x=\frac{1}{4}\pi+k\cdot\frac{1}{2}\pi.$
(de oplossingen in een schets op de rand van de eenheidscirkel uitzetten geeft ook snel de uiteindelijke oplossing)

24a $2\cos^2(\frac{1}{2}x)=1$
$\cos^2(\frac{1}{2}x)=\frac{1}{2}$
$\cos(\frac{1}{2}x)=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{2}\sqrt{2}$
$\frac{1}{2}x=\frac{1}{4}\pi+k\cdot\frac{1}{2}\pi$
$x=\frac{1}{2}\pi+k\cdot\pi.$

24b $4\sin^2(x-\frac{1}{6}\pi)=1$
$\sin^2(x-\frac{1}{6}\pi)=\frac{1}{4}$
$\sin(x-\frac{1}{6}\pi)=\pm\frac{1}{2}$
$x-\frac{1}{6}\pi=\frac{1}{6}\pi+k\cdot 2\pi$ of $x-\frac{1}{6}\pi=\frac{5}{6}\pi+k\cdot 2\pi$ of $x-\frac{1}{6}\pi=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x-\frac{1}{6}\pi=1\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{1}{3}\pi+k\cdot 2\pi$ of $x=\pi+k\cdot 2\pi$ of $x=k\cdot 2\pi$ of $x=1\frac{1}{3}\pi+k\cdot 2\pi$
$x=\frac{1}{3}\pi+k\cdot\pi$ of $x=\pi+k\cdot\pi.$

24c $4\cos^2(x+\frac{1}{4}\pi)=3$
$\cos^2(x+\frac{1}{4}\pi)=\frac{3}{4}$
$\cos(x+\frac{1}{4}\pi)=\pm\sqrt{\frac{3}{4}}=\pm\frac{1}{2}\sqrt{3}$
$x+\frac{1}{4}\pi=\frac{1}{6}\pi+k\cdot 2\pi$ of $x+\frac{1}{4}\pi=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x+\frac{1}{4}\pi=\frac{5}{6}\pi+k\cdot 2\pi$ of $x+\frac{1}{4}\pi=-\frac{5}{6}\pi+k\cdot 2\pi$
$x=-\frac{1}{12}\pi+k\cdot 2\pi$ of $x=-\frac{5}{12}\pi+k\cdot 2\pi$ of $x=\frac{7}{12}\pi+k\cdot 2\pi$ of $x=-\frac{13}{12}\pi+k\cdot 2\pi$
$x=-\frac{1}{12}\pi+k\cdot\pi$ of $x=-\frac{5}{12}\pi+k\cdot\pi.$

24d $4\sin^3(x)-\sin(x)=0$
$\sin(x)\cdot(4\sin^2(x)-1)=0$
$\sin(x)=0$ of $4\sin^2(x)=1$
$\sin(x)=0$ of $\sin^2(x)=\frac{1}{4}$
$\sin(x)=0$ of $\sin(x)=\pm\frac{1}{2}$
$x=k\cdot\pi$ of $x=\frac{1}{6}\pi+k\cdot 2\pi$ of $x=\frac{5}{6}\pi+k\cdot 2\pi$ of $x=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x=\frac{7}{6}\pi+k\cdot 2\pi$
$x=k\cdot\pi$ of $x=\frac{1}{6}\pi+k\cdot\pi$ of $x=\frac{5}{6}\pi+k\cdot\pi.$

24e $2\cos^2(x)=\cos(x)+1$
$2\cos^2(x)-\cos(x)-1=0$
$\cos^2(x)-\frac{1}{2}\cos(x)-\frac{1}{2}=0$ even proberen geeft: $(\cos(x)-\ldots)\cdot(\cos(x)+\ldots)=0$ (anders *abc*-formule)
$(\cos(x)-1)\cdot(\cos(x)+\frac{1}{2})=0$
$\cos(x)=1$ of $\cos(x)=-\frac{1}{2}$
$x=k\cdot 2\pi$ of $x=\frac{2}{3}\pi+k\cdot 2\pi$ of $x=-\frac{2}{3}\pi+k\cdot 2\pi$
$x=k\cdot\frac{2}{3}\pi.$

24f $\cos^2(x)-\cos(x)+\frac{1}{4}=0$ even proberen geeft: $(\cos(x)-\ldots)\cdot(\cos(x)-\ldots)=0$ (anders *abc*-formule)
$(\cos(x)-\frac{1}{2})\cdot(\cos(x)-\frac{1}{2})=0$
$\cos(x)=\frac{1}{2}$
$x=\frac{1}{3}\pi+k\cdot 2\pi$ of $x=-\frac{1}{3}\pi+k\cdot 2\pi.$

25a $\sin(\frac{1}{2}\pi x) = \frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x = \frac{1}{3}\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3} + k \cdot 4$ of $x = \frac{4}{3} + k \cdot 4.$

x op $[0, 10]$ geeft

$x = \frac{2}{3}$ of $x = 4\frac{2}{3}$ of $x = 8\frac{2}{3}$ of $x = 1\frac{1}{3}$ of $x = 5\frac{1}{3}$ of $x = 9\frac{1}{3}.$

25b $\cos(\frac{1}{3}\pi x) = -\frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$ of $\frac{1}{3}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{15}{6} + k \cdot 6$ of $x = -\frac{15}{6} + k \cdot 6.$

x op $[0, 10]$ geeft

$x = 2\frac{1}{2}$ of $x = 8\frac{1}{2}$ of $x = 3\frac{1}{2}$ of $x = 9\frac{1}{2}.$

25c $4\sin^2(\frac{1}{5}\pi x) = 1$

$\sin^2(\frac{1}{5}\pi x) = \frac{1}{4}$

$\sin(\frac{1}{5}\pi x) = \pm\frac{1}{2}$

$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = -\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = \frac{7}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6} + k \cdot 10$ of $x = \frac{25}{6} + k \cdot 10$ of $x = -\frac{5}{6} + k \cdot 10$ of $x = \frac{35}{6} + k \cdot 10.$

x op $[0, 10]$ geeft $x = \frac{5}{6}$ of $x = 4\frac{1}{6}$ of $x = 9\frac{1}{6}$ of $x = 5\frac{5}{6}.$

25d $2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) = 1$

$2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) - 1 = 0$

$\cos^2(0{,}1\pi x) + \frac{1}{2}\cos(0{,}1\pi x) - \frac{1}{2} = 0$ proberen geeft: $(\cos(0{,}1\pi x) + \ldots) \cdot (\cos(0{,}1\pi x) - \ldots) = 0$ (anders *abc*-formule)

$(\cos(0{,}1\pi x) + 1) \cdot (\cos(0{,}1\pi x) - \frac{1}{2}) = 0$

$\cos(0{,}1\pi x) = -1$ of $\cos(0{,}1\pi x) = \frac{1}{2}$

$0{,}1\pi x = \pi + k \cdot 2\pi$ of $0{,}1\pi x = \frac{1}{3}\pi + k \cdot 2\pi$ of $0{,}1\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = 10 + k \cdot 20$ of $x = \frac{10}{3} + k \cdot 20$ of $x = -\frac{10}{3} + k \cdot 20.$

x op $[0, 10]$ geeft $x = 10$ of $x = 3\frac{1}{3}.$

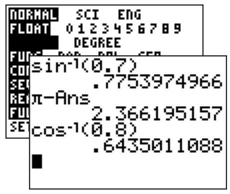
26a $\sin(x) = 0{,}7$

$x \approx 0{,}775 + k \cdot 2\pi$ of $x \approx \pi - 0{,}775 + k \cdot 2\pi$

$x \approx 0{,}775 + k \cdot 2\pi$ of $x \approx 2{,}366 + k \cdot 2\pi.$

26b $\cos(x) = 0{,}8$

$x \approx 0{,}644 + k \cdot 2\pi$ of $x \approx -0{,}644 + k \cdot 2\pi.$



27a $\sin(x) = -0{,}85$

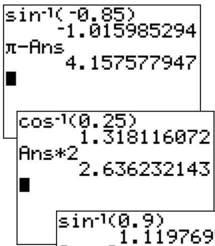
$x \approx -1{,}016 + k \cdot 2\pi$ of $x \approx \pi - -1{,}016 + k \cdot 2\pi$

$x \approx -1{,}016 + k \cdot 2\pi$ of $x \approx 4{,}158 + k \cdot 2\pi.$

27b $\cos(\frac{1}{2}x) = 0{,}25$

$\frac{1}{2}x \approx 1{,}318 + k \cdot 2\pi$ of $\frac{1}{2}x \approx -1{,}318 + k \cdot 2\pi$

$x \approx 2{,}636 + k \cdot 4\pi$ of $x \approx -2{,}636 + k \cdot 4\pi.$



27c $\sin(x + 2) = 0{,}9$

$x + 2 \approx 1{,}120 + k \cdot 2\pi$ of $x + 2 \approx \pi - 1{,}120 + k \cdot 2\pi$

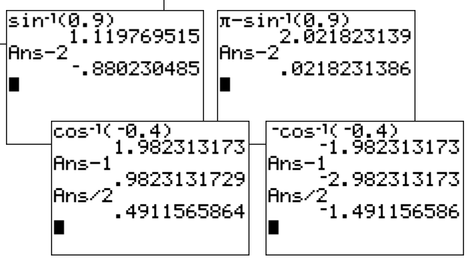
$x \approx -0{,}880 + k \cdot 2\pi$ of $x \approx 0{,}022 + k \cdot 2\pi.$

27d $\cos(2x + 1) = -0{,}4$

$2x + 1 \approx 1{,}982 + k \cdot 2\pi$ of $2x + 1 \approx -1{,}982 + k \cdot 2\pi$

$2x \approx 0{,}982 + k \cdot 2\pi$ of $2x \approx -2{,}982 + k \cdot 2\pi$

$x \approx 0{,}491 + k \cdot \pi$ of $x \approx -1{,}491 + k \cdot \pi.$



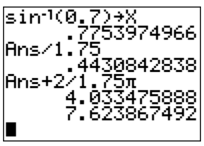
28a $2\sin(1{,}75x) = 1{,}4$

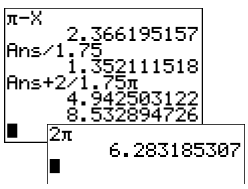
$\sin(1{,}75x) = 0{,}7$

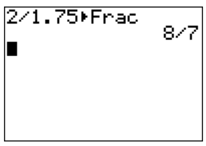
$1{,}75x \approx 0{,}775 + k \cdot 2\pi$ of $1{,}75x \approx \pi - 0{,}775 + k \cdot 2\pi$

$x \approx 0{,}443 + k \cdot \frac{2}{1{,}75}\pi$ of $x \approx 1{,}352 + k \cdot \frac{2}{1{,}75}\pi.$

x op $[0, 2\pi]$ geeft $x \approx 0{,}443$ of $x \approx 4{,}033$ of $x \approx 1{,}352$ of $x \approx 4{,}943.$





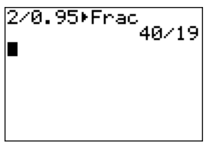


28b $\cos^2(0{,}95x) = 0{,}86$

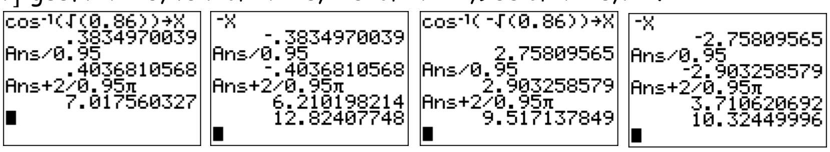
$\cos(0{,}95x) = \pm\sqrt{0{,}86}$

$0{,}95x \approx 0{,}383 + k \cdot 2\pi$ of $0{,}95x \approx -0{,}383 + k \cdot 2\pi$ of $0{,}95x \approx 2{,}758 + k \cdot 2\pi$ of $0{,}95x \approx -2{,}758 + k \cdot 2\pi$

$x \approx 0{,}404 + k \cdot \frac{2}{0{,}95}\pi$ of $x \approx -0{,}404 + k \cdot \frac{2}{0{,}95}\pi$ of $x \approx 2{,}903 + k \cdot \frac{2}{0{,}95}\pi$ of $x \approx -2{,}903 + k \cdot \frac{2}{0{,}95}\pi.$



x op $[0, 2\pi]$ geeft $x \approx 0{,}404$ of $x \approx 6{,}210$ of $x \approx 2{,}903$ of $x \approx 3{,}711.$



29a $\sin(3x) = \sin(\frac{1}{6}\pi)$
$3x = \frac{1}{6}\pi + k \cdot 2\pi$ of $3x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi.$

29b $\cos(3x) = \cos(\frac{1}{6}\pi)$
$3x = \frac{1}{6}\pi + k \cdot 2\pi$ of $3x = -\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$ of $x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi.$

30a $\sin(x+1) = \sin(2x+3)$
$x + 1 = 2x + 3 + k \cdot 2\pi$ of $x + 1 = \pi - (2x+3) + k \cdot 2\pi$
$x + 1 = 2x + 3 + k \cdot 2\pi$ of $x + 1 = \pi - 2x - 3 + k \cdot 2\pi$
$-x = 2 + k \cdot 2\pi$ of $3x = \pi - 4 + k \cdot 2\pi$
$x = -2 + k \cdot 2\pi$ of $x = \frac{1}{3}\pi - \frac{4}{3} + k \cdot \frac{2}{3}\pi.$

30b $\cos(2x-1) = \cos(x+1)$
$2x - 1 = x + 1 + k \cdot 2\pi$ of $2x - 1 = -(x+1) + k \cdot 2\pi$
$2x - 1 = x + 1 + k \cdot 2\pi$ of $2x - 1 = -x - 1 + k \cdot 2\pi$
$x = 2 + k \cdot 2\pi$ of $3x = k \cdot 2\pi$
$x = 2 + k \cdot 2\pi$ of $x = k \cdot \frac{2}{3}\pi.$

30c $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$
$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \frac{2}{3}\pi - x + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi$ of $3x = \frac{7}{6}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi$ of $x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi.$

30d $\cos(x - \frac{1}{3}\pi) = \cos(2x)$
$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi$ of $x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$
$-x = \frac{1}{3}\pi + k \cdot 2\pi$ of $3x = \frac{1}{3}\pi + k \cdot 2\pi$
$x = -\frac{1}{3}\pi + k \cdot 2\pi$ of $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi.$

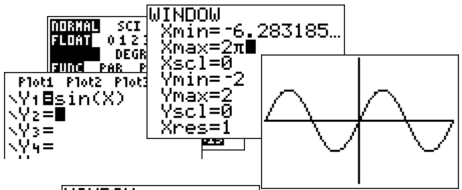
30e $\sin(2\pi x) = \sin(\pi(x-1))$
$2\pi x = \pi(x-1) + k \cdot 2\pi$ of $2\pi x = \pi - \pi(x-1) + k \cdot 2\pi$
$2\pi x = \pi x - \pi + k \cdot 2\pi$ of $2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$
$\pi x = -\pi + k \cdot 2\pi$ of $3\pi x = 2\pi + k \cdot 2\pi$
$x = -1 + k \cdot 2$ of $x = \frac{2}{3} + k \cdot \frac{2}{3}$
$x = -1 + k \cdot 2$ of $x = k \cdot \frac{2}{3}.$

30f $\cos(\frac{1}{2}\pi x) = \cos(\pi(x-2))$
$\frac{1}{2}\pi x = \pi(x-2) + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi(x-2) + k \cdot 2\pi$
$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
$-\frac{1}{2}\pi x = -2\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
$x = 4 + k \cdot 4$ of $\frac{3}{2}\pi x = 2\pi + k \cdot 2\pi$
$x = k \cdot 4$ of $x = \frac{4}{3} + k \cdot \frac{4}{3}$
$x = k \cdot 4$ of $x = k \cdot \frac{4}{3}$
$x = k \cdot \frac{4}{3}.$

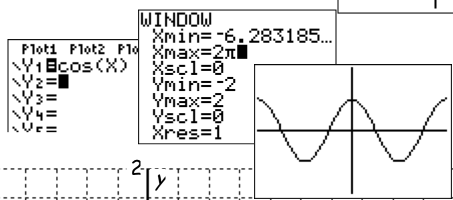
31a $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$
$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi$ of $2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$
$x = \frac{7}{12}\pi + k \cdot 2\pi$ of $2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$
$x = \frac{7}{12}\pi + k \cdot 2\pi$ of $3x = \frac{13}{12}\pi + k \cdot 2\pi$
$x = \frac{7}{12}\pi + k \cdot 2\pi$ of $x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi.$
x op $[0, 2\pi]$ geeft $x = \frac{7}{12}\pi$ of $x = \frac{13}{36}\pi$ of $x = \frac{37}{36}\pi$ of $x = \frac{61}{36}\pi.$

31b $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$
$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi$ of $3x + \frac{1}{2}\pi = -(2x - \frac{1}{4}\pi) + k \cdot 2\pi$
$x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$
$x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $5x = -\frac{1}{4}\pi + k \cdot 2\pi$
$x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi.$
x op $[0, 2\pi]$ geeft $x = \frac{5}{4}\pi$ of $x = \frac{7}{20}\pi$ of $x = \frac{15}{20}\pi$ of $x = \frac{23}{20}\pi$ of $x = \frac{31}{20}\pi$ of $x = \frac{39}{20}\pi.$

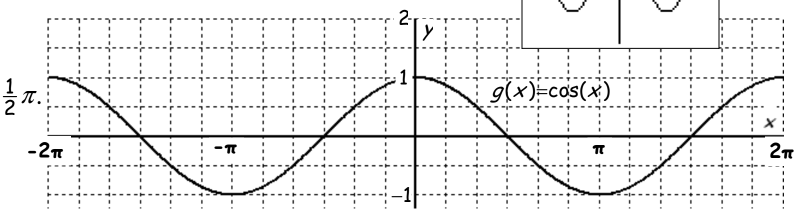
32a Zie de plot hiernaast. (denk ook aan de eenheidscirkel)
32b De toppen zijn: $(-1\frac{1}{2}\pi, 1)$, $(-\frac{1}{2}\pi, -1)$, $(\frac{1}{2}\pi, 1)$ en $(1\frac{1}{2}\pi, -1)$.
32c Snijpunten met de x-as: $(-2\pi, 0)$, $(-\pi, 0)$, $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.



33a Zie de plot hiernaast. (denk ook aan de eenheidscirkel)
33b De toppen zijn: $(-2\pi, 1)$, $(-\pi, -1)$, $(0, 1)$, $(\pi, -1)$ en $(2\pi, 1)$.



33c $g(x) = \cos x = 0 \Rightarrow x = -1\frac{1}{2}\pi$ of $x = -\frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$ of $x = 1\frac{1}{2}\pi.$
OF: $\cos x = 0 \Rightarrow x = \frac{1}{2}\pi + k \cdot \pi$
Dus op domein $[-2\pi, 2\pi]$ $\cos x = 0$ geeft
$x = -1\frac{1}{2}\pi$ of $x = -\frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$ of $x = 1\frac{1}{2}\pi.$
33d Zie de grafiek hiernaast.

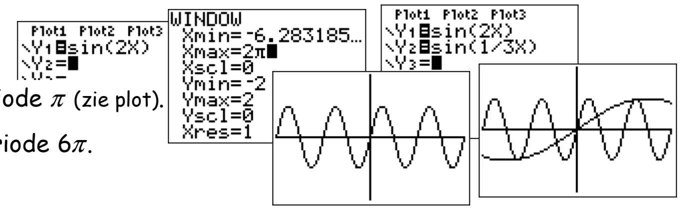


34a $f(x) = \sin(x) \xrightarrow{\text{translatie } (0,2)} g(x) = \sin(x) + 2.$
evenwichtswaarde 0 $\Rightarrow$ evenwichtswaarde 2

34b $f(x) = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} h(x) = \sin(x - \frac{1}{3}\pi).$ Nulpunten: $h(x) = \sin(x - \frac{1}{3}\pi) = 0$
$x - \frac{1}{3}\pi = 0 + k \cdot \pi$
$x = \frac{1}{3}\pi + k \cdot \pi$

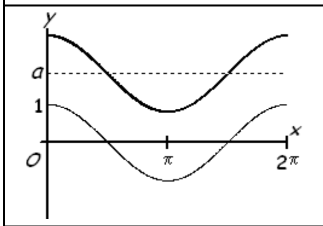
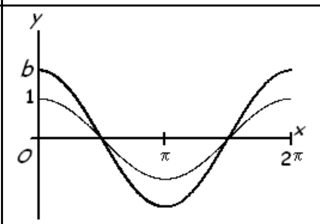
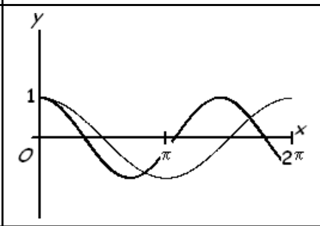
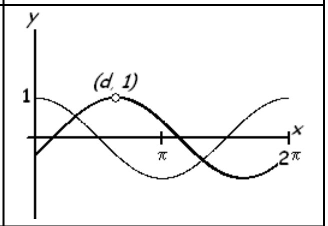
34c $f(x) = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 4} k(x) = 4\sin(x).$
amplitude 1 $\Rightarrow$ amplitude 4

35a Zie de plot hiernaast.
35b $f(x) = \sin(2x)$ heeft periode π (zie plot).
35c $g(x) = \sin(\frac{1}{3}x)$ heeft periode 6π.



$\sin(2x) = 1$
$2x = \frac{1}{2}\pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \pi \Rightarrow$ maxima om de π
Dus sin(2x) heeft periode π

36 Zie het schema voor de cosinus hieronder. (de oorspronkelijke grafiek van $y = \cos(x)$ is steeds iets dunner getekend)

translatie $(0, a)$	vermenigvuldiging x-as, b	vermenigvuldiging y-as, $\frac{1}{c}$	translatie $(d, 0)$
tel a op bij de functiewaarde	verm. de funtiewaarde met b	vervang x door cx	vervang x door $x - d$
			
$y = a + \cos(x)$ evenwichtsstand is a	$y = b\cos(x)$ amplitude is b	$y = \cos(cx)$ periode is $\frac{2\pi}{c}$	$y = \cos(x - d)$ punt op de y-as komt bij $x = d$

37a $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2\sin(x) \xrightarrow{\text{translatie } (-3, 0)} f(x) = 2\sin(x + 3).$ (of in omgekeerde volgorde)

37b $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } \frac{1}{3}} y = \frac{1}{3}\sin(x) \xrightarrow{\text{translatie } (0, \frac{1}{5})} g(x) = \frac{1}{3}\sin(x) + \frac{1}{5}.$ (in deze volgorde)

37c $y = \cos(x) \xrightarrow{\text{translatie } (12, 0)} y = \cos(x - 12) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} h(x) = \cos(3x - 12).$ (in deze volgorde)

37d $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 1\frac{1}{2}} y = 1\frac{1}{2}\cos(x) \xrightarrow{\text{verm. } y\text{-as, } 4} j(x) = 1\frac{1}{2}\cos(\frac{1}{4}x).$ (of in omgekeerde volgorde)

38a $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 1{,}2} y = 1{,}2\cos(x) \xrightarrow{\text{translatie } (\frac{1}{6}\pi, 5)} f(x) = 1{,}2\cos(x - \frac{1}{6}\pi) + 5.$ (in deze volgorde)

38b $y = \sin(x) \xrightarrow{\text{verm. } y\text{-as, } 5} y = \sin(\frac{1}{5}x) \xrightarrow{\text{translatie } (-\frac{1}{3}\pi; 0{,}4)} g(x) = \sin(\frac{1}{5}(x + \frac{1}{3}\pi)) + 0{,}4.$ (in deze volgorde)

38c $y = \cos(x) \xrightarrow{\text{translatie } (-4{,}2; 0)} y = \cos(x + 4{,}2) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = \cos(3x + 4{,}2)$ (in deze volgorde)
$y = \cos(3x + 4{,}2) \xrightarrow{\text{verm. } x\text{-as, } 0{,}29} h(x) = 0{,}29\cos(3x + 4{,}2)$ (deze laatste vermenigvuldiging mag ook eerder staan).

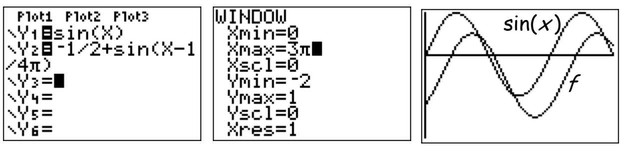
38d $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2\sin(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = 2\sin(3x)$ (of in omgekeerde volgorde en daarna)
$y = 2\sin(3x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi, -0{,}8)} j(x) = 2\sin(3(x - \frac{1}{2}\pi)) - 0{,}8.$ (deze translatie moet als laatste staan)

39 $y = \sin(x) \xrightarrow{\text{verm. } y\text{-as, } 3} y = \sin(\frac{1}{3}x) \xrightarrow{\text{translatie } (4; -1{,}5)} f(x) = \sin(\frac{1}{3}(x - 4)) - 1{,}5.$

40a $y = \cos(x) \xrightarrow{\text{transl. } (\frac{1}{4}\pi, 4)} y = \cos(x - \frac{1}{4}\pi) + 4 \xrightarrow{\text{verm. } x\text{-as, } 3} f(x) = 3(\cos(x - \frac{1}{4}\pi) + 4) = 3\cos(x - \frac{1}{4}\pi) + 12.$

40b $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{translatie } (\frac{1}{4}\pi; 4)} g(x) = 3\cos(x - \frac{1}{4}\pi) + 4.$

41a $y = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{4}\pi, -\frac{1}{2})} f(x) = \sin(x - \frac{1}{4}\pi) - \frac{1}{2}.$
Maak een schets van de plot van f hiernaast.



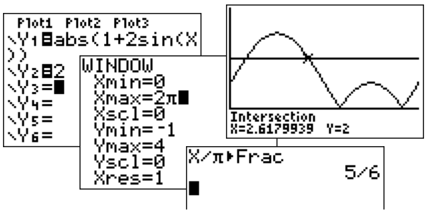
41b Beginpunt (0, 0) (waar $y=\sin(x)$ stijgend door de evenwichtswaarde gaat) ligt door de translatie in $(\frac{1}{4}\pi,\ -\frac{1}{2})$; periode blijft 2π, dus $f(x)=-\frac{1}{2}+\sin(x-\frac{1}{4}\pi)$ snijdt evenwichtswaarde in $(\frac{1}{4}\pi,\ -\frac{1}{2})$, $(1\frac{1}{4}\pi,\ -\frac{1}{2})$ en $(2\frac{1}{4}\pi,\ -\frac{1}{2})$.

41c De toppen (een kwart periode rechts van de snijpunten met de evenwichtswaarde) zijn: $(\frac{3}{4}\pi,\ \frac{1}{2})$, $(1\frac{3}{4}\pi,\ -1\frac{1}{2})$ en $(2\frac{3}{4}\pi,\ \frac{1}{2})$.

41d $f(x)=-\frac{1}{2}+\sin(x-\frac{1}{4}\pi)=0$
$\sin(x-\frac{1}{4}\pi)=\frac{1}{2}$
$x-\frac{1}{4}\pi=\frac{1}{6}\pi+k\cdot 2\pi$ of $x-\frac{1}{4}\pi=\pi-\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{5}{12}\pi+k\cdot 2\pi$ of $x=\frac{13}{12}\pi+k\cdot 2\pi$.
Op $[0,\ 3\pi]$ geeft: $x=\frac{5}{12}\pi$ of $x=2\frac{5}{12}\pi$ of $x=1\frac{1}{12}\pi$.
Dus $x_A=\frac{5}{12}\pi$, $x_B=1\frac{1}{12}\pi$ en $x_C=2\frac{5}{12}\pi$.
$AB=x_B-x_A=1\frac{1}{12}\pi-\frac{5}{12}\pi=\frac{8}{12}\pi$.

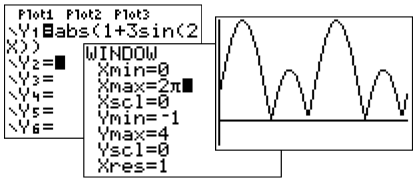
41e $f(x)=-\frac{1}{2}+\sin(x-\frac{1}{4}\pi)=-1$
$\sin(x-\frac{1}{4}\pi)=-\frac{1}{2}$
$x-\frac{1}{4}\pi=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x-\frac{1}{4}\pi=\pi--\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{1}{12}\pi+k\cdot 2\pi$ of $x=\frac{17}{12}\pi+k\cdot 2\pi$.
Op $[0,\ 3\pi]$ geeft: $x=\frac{1}{12}\pi$ of $x=2\frac{1}{12}\pi$ of $x=1\frac{5}{12}\pi$.
Gebruik vervolgens de plot om daarin af te lezen:
$f(x)\geq -1$ voor $\frac{1}{12}\pi\leq x\leq 1\frac{5}{12}\pi$ of $2\frac{1}{12}\pi\leq x\leq 3\pi$.

42 $f(x)=|1+2\sin(x)|=2$
$1+2\sin(x)=2$ of $1+2\sin(x)=-2$
$2\sin(x)=1$ of $2\sin(x)=-3$
$\sin(x)=\frac{1}{2}$ of $\sin(x)=-1\frac{1}{2}$ (kan niet)
$x=\frac{1}{6}\pi+k\cdot 2\pi$ of $x=\pi-\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{1}{6}\pi+k\cdot 2\pi$ of $x=\frac{5}{6}\pi+k\cdot 2\pi$.
Op $[0,\ 2\pi]$ geeft: $x=\frac{1}{6}\pi$ of $x=\frac{5}{6}\pi$.
Gebruik nu een plot om af te lezen:
$f(x)\geq 2$ voor $\frac{1}{6}\pi\leq x\leq\frac{5}{6}\pi$.

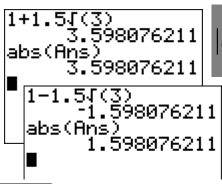


43a Maak een schets van de plot hiernaast. (de grafiek komt 4 keer aan de x-as)

43b $y=\sin(x)\xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y=\sin(2x)$.
De toppen van $y=\sin(2x)$ zijn $(\frac{1}{4}\pi,\ 1)$, $(\frac{3}{4}\pi,\ -1)$, $(1\frac{1}{4}\pi,\ 1)$ en $(1\frac{3}{4}\pi,\ -1)$.
De toppen van $f(x)$ zijn $(\frac{1}{4}\pi,\ 4)$, $(\frac{3}{4}\pi,\ 2)$, $(1\frac{1}{4}\pi,\ 4)$ en $(1\frac{3}{4}\pi,\ 2)$.



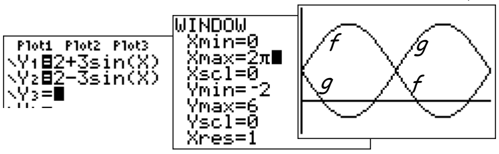
43c $f(\frac{1}{6}\pi)=|1+3\sin(2\cdot\frac{1}{6}\pi)|=|1+3\sin(\frac{1}{3}\pi)|=|1+3\cdot\frac{1}{2}\sqrt{3}|=|1+1\frac{1}{2}\sqrt{3}|=1+1\frac{1}{2}\sqrt{3}$.
$f(\frac{1}{3}\pi)=|1+3\sin(2\cdot\frac{1}{3}\pi)|=|1+3\sin(\frac{2}{3}\pi)|=|1+3\cdot\frac{1}{2}\sqrt{3}|=|1+1\frac{1}{2}\sqrt{3}|=1+1\frac{1}{2}\sqrt{3}$.
$f(\frac{2}{3}\pi)=|1+3\sin(2\cdot\frac{2}{3}\pi)|=|1+3\sin(\frac{4}{3}\pi)|=|1+3\cdot-\frac{1}{2}\sqrt{3}|=|1-1\frac{1}{2}\sqrt{3}|=-1+1\frac{1}{2}\sqrt{3}$.
$f(\frac{5}{6}\pi)=|1+3\sin(2\cdot\frac{5}{6}\pi)|=|1+3\sin(\frac{5}{3}\pi)|=|1+3\cdot-\frac{1}{2}\sqrt{3}|=|1-1\frac{1}{2}\sqrt{3}|=-1+1\frac{1}{2}\sqrt{3}$.



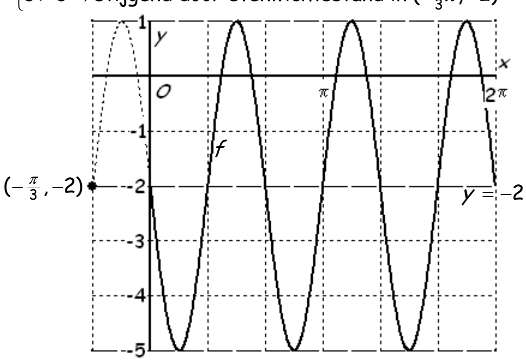
$|a|=\begin{cases} a \text{ voor } a\geq 0\\ -a \text{ voor } a<0\end{cases}$

44a Zie de plot hiernaast.

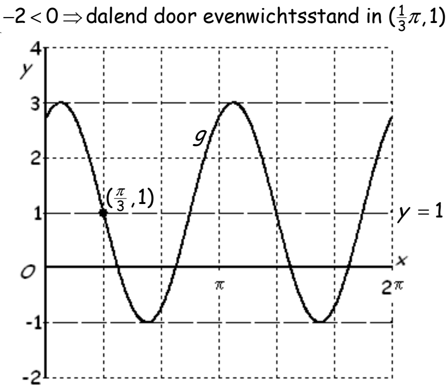
44b De amplitude is bij beide grafieken 3.



45a $f(x)=-2+3\sin(3x+\pi)=-2+3\sin(3(x+\frac{1}{3}\pi)$.
$\begin{cases}\text{evenwichtsstand } -2\\ \text{amplitude } 3\\ \text{periode } \frac{2\pi}{3}=\frac{2}{3}\pi\\ 3>0\Rightarrow\text{stijgend door evenwichtsstand in } (-\frac{1}{3}\pi,-2)\end{cases}$

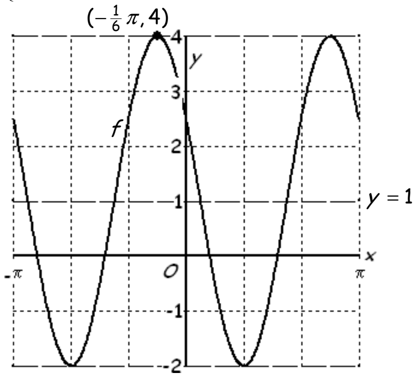


45b $g(x)=1-2\sin(2x-\frac{2}{3}\pi)=1-2\sin(2(x-\frac{1}{3}\pi)$.
$\begin{cases}\text{evenwichtsstand } 1\\ \text{amplitude } 2\\ \text{periode } \frac{2\pi}{2}=\pi\\ -2<0\Rightarrow\text{dalend door evenwichtsstand in } (\frac{1}{3}\pi,1)\end{cases}$



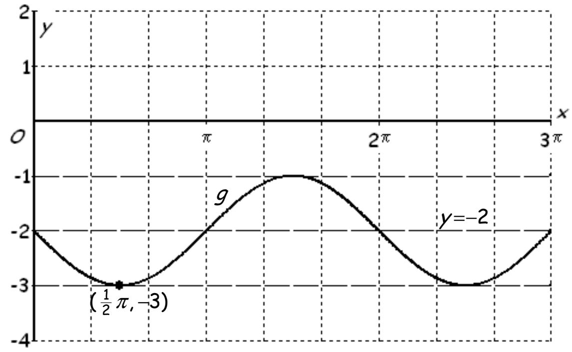
46a $f(x) = 1 + 3\cos(2x + \frac{1}{3}\pi) = 1 + 3\cos(2(x + \frac{1}{6}\pi))$.

evenwichtsstand 1
amplitude 3
periode $\frac{2\pi}{2} = \pi$
$3 > 0 \Rightarrow$ beginpunt $(-\frac{1}{6}\pi, 4)$ is hoogste punt



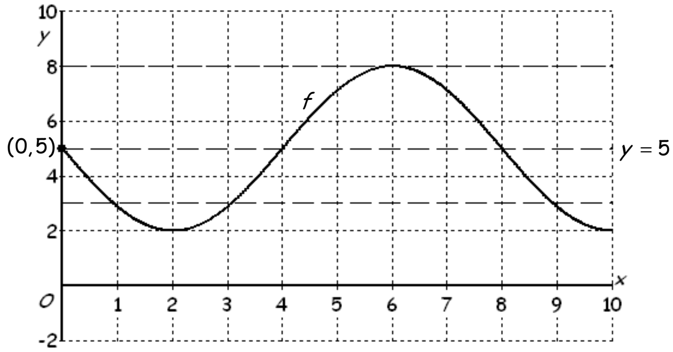
46b $g(x) = -2 - \cos(x - \frac{1}{2}\pi)$.

evenwichtsstand -2
amplitude 1
periode $\frac{2\pi}{1} = 2\pi$
$-1 < 0 \Rightarrow$ beginpunt $(\frac{1}{2}\pi, -3)$ is laagste punt



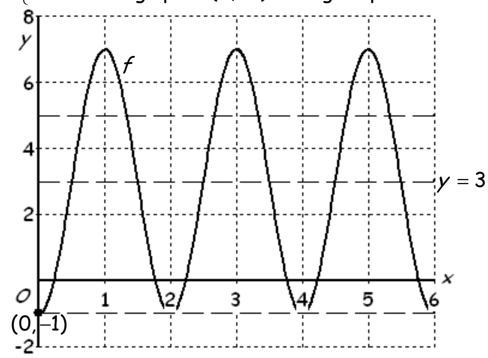
47 $f(x) = 5 - 3\sin(\frac{1}{4}\pi x)$.

evenwichtsstand 5
amplitude 3
periode $\frac{2\pi}{\frac{1}{4}\pi} = 8$
$-3 < 0 \Rightarrow$ dalend door evenwichtsstand in $(0, 5)$



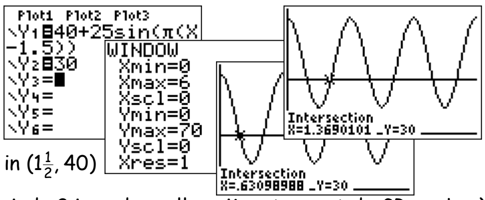
48 $f(x) = 3 - 4\cos(\pi x)$.

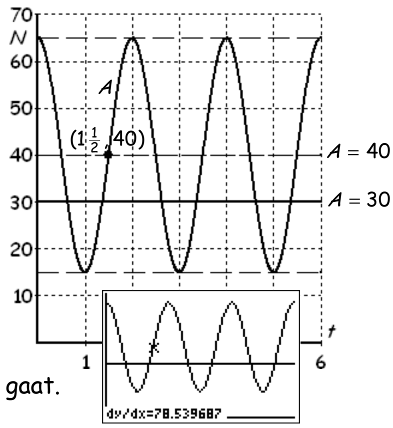
evenwichtsstand 3
amplitude 4
periode $\frac{2\pi}{\pi} = 2$
$-4 < 0 \Rightarrow$ beginpunt $(0, -1)$ is laagste punt



49a $A = 40 + 25\sin(\pi(t - 1\frac{1}{2}))$.

evenwichtsstand 40
amplitude 25
periode $\frac{2\pi}{\pi} = 2$
$25 > 0 \Rightarrow$ stijgend door evenwichtsstand in $(1\frac{1}{2}, 40)$



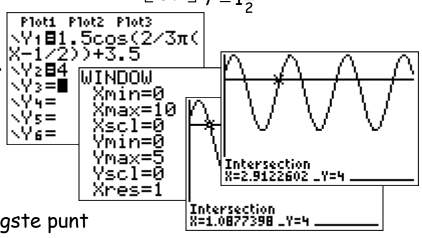


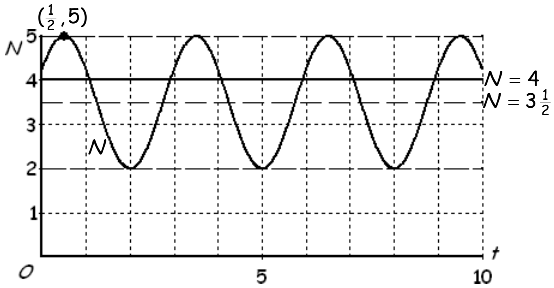
49b $A = 30$ (intersect) $\Rightarrow$ (bedenk dat de periode 2 is anders alle snijpunten met de GR zoeken)
$t \approx 0{,}63$ of $t \approx 1{,}37$ of $t \approx 2{,}63$ of $t \approx 3{,}37$ of $t \approx 4{,}63$ of $t \approx 5{,}37$.
Met gebruik van de plot (of de grafiek) vind je:
$A < 30$ op $[0, 6]$ voor $0{,}63 < t < 1{,}37$ of $2{,}63 < t < 3{,}37$ of $4{,}63 < t < 5{,}37$.

49c Maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat.
Dus in $(1\frac{1}{2}, 40) \Rightarrow$ de grootste helling is $\left[\frac{dA}{dt}\right]_{t = 1\frac{1}{2}} \approx 78{,}5$.

50a $N = 1\frac{1}{2}\cos(\frac{2}{3}\pi(t - \frac{1}{2})) + 3\frac{1}{2}$.

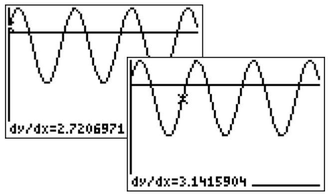
evenwichtsstand $3\frac{1}{2}$
amplitude $1\frac{1}{2}$
periode $\frac{2\pi}{\frac{2}{3}\pi} = 3$
$1\frac{1}{2} > 0 \Rightarrow$ beginpunt $(\frac{1}{2}, 5)$ is hoogste punt





50b $A = 4$ (intersect) $\Rightarrow$ (bedenk dat de periode 3 is)
$t \approx 1{,}09$ of $t \approx 2{,}91$ of $t \approx 4{,}09$ of $t \approx 5{,}91$ of $t \approx 7{,}09$ of $t \approx 8{,}91$.
Met behulp van de grafiek vind je: $N > 4$ voor $0 \le t < 1{,}09$ of $2{,}91 < t < 4{,}09$ of $5{,}91 < t < 7{,}09$ of $8{,}91 < t \le 10$.

50c De helling in het snijpunt met de verticale as ($t = 0$) is $\left[\frac{dN}{dt}\right]_{t=0} \approx 2{,}72$.



50d Maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat.
Dit is een kwart periode voor het hoogste punt $\Rightarrow$ in $(\frac{1}{2} + \frac{3}{4} \cdot 3,\ 3\frac{1}{2})$ of in $(\frac{11}{4},\ 3\frac{1}{2})$.
De grootste helling is $\left[\frac{dN}{dt}\right]_{t=2\frac{3}{4}} \approx 3{,}1$.

51a $j(x) = \sin(2x) - \frac{1}{2}$. (de enige met amplitude 1 of evenwichtsstand $-\frac{1}{2}$)

51b $f(x) = 1\frac{1}{2}\sin(2x)$. (de enige met amplitude $1\frac{1}{2}$ én evenwichtsstand 0)

51c $g(x) = 1\frac{1}{2}\sin(x) + 1$. (de enige met evenwichtsstand 1)

51d $h(x) = 2\sin(1\frac{1}{2}x)$. (de enige met amplitude 2)

52a $y = a + b\sin(c(x - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) $= \frac{50 + -10}{2} = \frac{40}{2} = 20$; b (= amplitude) $= 50 - 20 = 30$;
c (= $\frac{2\pi}{\text{periode}}$) $= \frac{2\pi}{50} = \frac{1}{25}\pi$ en $d = 0$ (de sinus gaat stijgend door de evenwichtsstand voor $x = 0$) $\Rightarrow y = 20 + 30\sin(\frac{1}{25}\pi x)$.

52b $y = a + b\sin(c(x - d))$ met $d = 25$ (sinus gaat dalend door de evenwichtsstand voor $x = 25$) $\Rightarrow y = 20 - 30\sin(\frac{1}{25}\pi(x - 25))$.

52c $y = a + b\cos(c(x - d))$ met $d = 12\frac{1}{2}$ (de cosinus heeft hoogste punt voor $x = 12\frac{1}{2}$) $\Rightarrow y = 20 + 30\cos(\frac{1}{25}\pi(x - 12\frac{1}{2}))$.

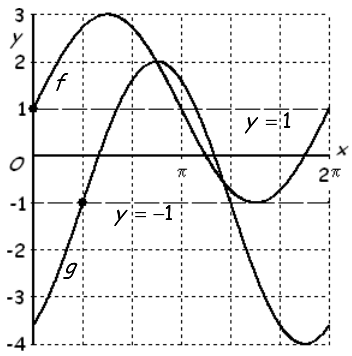
52d $y = a + b\cos(c(x - d))$ met $d = 37\frac{1}{2}$ (de cosinus heeft laagste punt voor $x = 37\frac{1}{2}$) $\Rightarrow y = 20 - 30\cos(\frac{1}{25}\pi(x - 37\frac{1}{2}))$.

53a $N = a + b\sin(c(t - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) $= \frac{100 + -220}{2} = \frac{-120}{2} = -60$; b (= amplitude) $= 100 + 60 = 160$;
c (= $\frac{2\pi}{\text{periode}}$) $= \frac{2\pi}{6{,}8} = \frac{5}{17}\pi$ en $d = 4$ (sinus gaat stijgend door de evenwichtsstand voor $t = 4$) $\Rightarrow N = -60 + 160\sin(\frac{5}{17}\pi(t - 4))$.

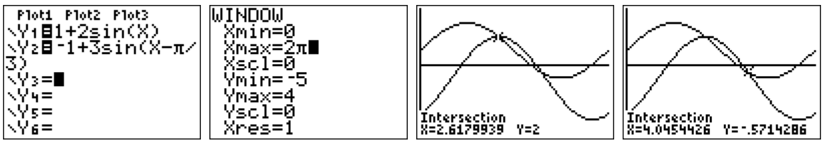
53b $N = a + b\cos(c(t - d))$ met $d = 5{,}7$ (de cosinus heeft hoogste punt voor $t = 5{,}7$) $\Rightarrow N = -60 + 160\cos(\frac{5}{17}\pi(t - 5{,}7))$.

54a $f(x) = 1 + 2\sin(x)$.
- evenwichtsstand 1
- amplitude 2
- periode $\frac{2\pi}{1} = 2\pi$
- $2 > 0 \Rightarrow$ stijgend door evenw.stand voor $x = 0$

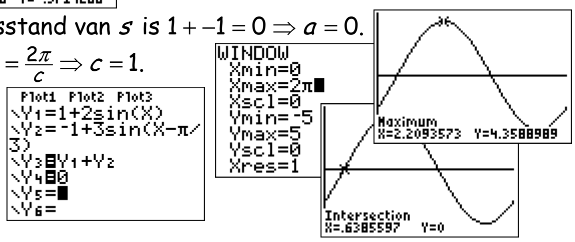
$g(x) = -1 + 3\sin(x - \frac{1}{3}\pi)$
- evenwichtsstand -1
- amplitude 3
- periode $\frac{2\pi}{1} = 2\pi$
- $3 > 0 \Rightarrow$ stijgend door evenwichtsstand voor $x = \frac{1}{3}\pi$



54b $f(x) = g(x)$ (intersect) $\Rightarrow x \approx 2{,}62$ of $x \approx 4{,}05$.
Dan met de grafiek: $f(x) > g(x)$ op $[0,\ 2\pi]$ voor $0 \le x < 2{,}62$ of $4{,}05 < x \le 2\pi$.

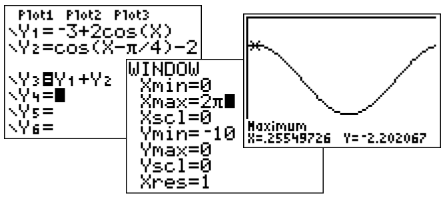


54c De evenwichtsstand van f is 1 en die van g is $-1 \Rightarrow$ evenwichtsstand van s is $1 + -1 = 0 \Rightarrow a = 0$.
De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van s is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1$.

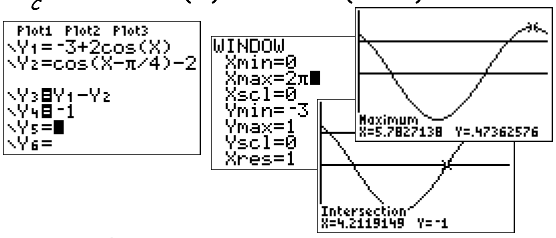


54d $s(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).
De evenwichtsstand van s is 0 en de periode is 2π.
Dus $s(x) = b\sin(x - d)$ met $b \approx 4{,}36$ (optie maximum).
(b = amplitude = maximum − evenwichtsstand = 4,36 − 0 = 4,36)
$s(x) = 0$ (intersect) $\Rightarrow x \approx 0{,}64 = d$.
(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand) Dus $s(x) = 4{,}36\sin(x - 0{,}64)$.

55a De evenwichtsstand van f is -3 en die van g is $-2 \Rightarrow$ evenwichtsstand van s is $-3 + -2 = -5 \Rightarrow a = -5$.
De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van s is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1 \Rightarrow s(x) = -5 + b\cos(x - d)$.
$s(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).
Optie maximum geeft maximum $s(0{,}26) \approx -2{,}20 \Rightarrow b \approx 2{,}80$ en $d \approx 0{,}26$.
(b = amplitude = maximum − evenwichtsstand = −2,20 − −5 = −2,20 + 5 = 2,80)
(de cosinus heeft, als $b > 0$, maximum bij $x = d$)
Dus $s(x) = -5 + 2{,}80\cos(x - 0{,}26)$.

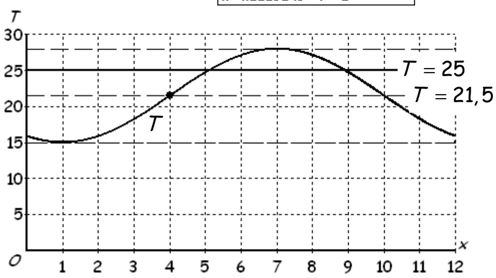


55b De evenwichtsstand van f is -3 en die van g is $-2 \Rightarrow$ evenwichtsstand van v is $-3 - -2 = -3 + 2 = -1 \Rightarrow a = -1$.
De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van v is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1 \Rightarrow v(x) = -1 + b\sin(x - d)$.
$v(x) = f(x) - g(x)$ (voer deze formule in op de GR; zet f en g uit).
Optie maximum geeft maximum $v(5,78) \approx 0,47 \Rightarrow b \approx 1,47$.
(b = amplitude = maximum − evenwichtsstand = $0,47 - -1 = 1,47$)
$v(x) = -1$ (intersect) $\Rightarrow x \approx 4,21 = d$.
(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand)
Dus $v(x) = -1 + 1,47\sin(x - 4,21)$.

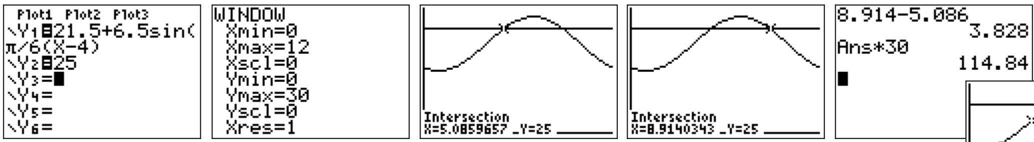


56a $T = 21,5 + 6,5\sin(\frac{1}{6}\pi(t-4))$.

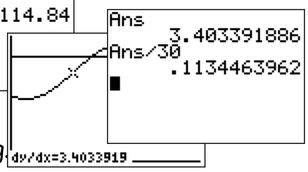
$\begin{cases} \text{evenwichtsstand } 21,5 \\ \text{amplitude } 6,5 \\ \text{periode } \frac{2\pi}{\frac{1}{6}\pi} = 12 \\ 6,5 > 0 \Rightarrow \text{stijgend door evenwichtsstand voor } t = 4 \end{cases}$



56b $T = 21,5 + 6,5\sin(\frac{1}{6}\pi(t-4)) = 25$ (intersect) $\Rightarrow x \approx 5,086$ of $x \approx 8,914$.
Dus gedurende $8,914 - 5,086 = 3,828$ maanden $\Rightarrow 3,828 \cdot 30 \approx 115$ dagen.



56c Sterkste stijging in het punt waar de grafiek stijgend door de evenwichtsstand gaat.
Dus voor $t = 4$. De sterkste stijging is $\left[\frac{dT}{dt}\right]_{t=4} \approx 3,40$ °C per maand $\Rightarrow 0,1$ °C per dag.



56d $a = 17,5$; $b = 17,5 - 15 = 2,5$; $c = \frac{2\pi}{12} = \frac{1}{6}\pi$ en $d = 2 + \frac{1}{4} \cdot 12 = 2 + 3 = 5$
(stijgend door de evenwichtsstand voor $t = d$, een kwart periode na de laagste temperatuur op 1 maart met $t = 2 \Rightarrow d = 2 + 3 = 5$).

Diagnostische toets

D1a $\sin(-270^\circ) = \sin(-270^\circ + 360^\circ) = \sin(90^\circ) = 1.$

D1b $\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{2}\sqrt{2}.$

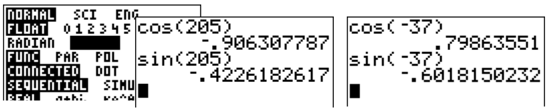
D1c $\cos 225^\circ = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}.$

D1d $\cos(-120^\circ) = \cos(-180^\circ + 60^\circ) = -\cos 60^\circ = -\frac{1}{2}.$

D1e $\sin 240^\circ = \sin(180^\circ + 60^\circ) = -\sin 60^\circ = -\frac{1}{2}\sqrt{3}.$

D1f $\cos 330^\circ = \cos(-30^\circ) = \cos 30^\circ = -\frac{1}{2}\sqrt{3}.$

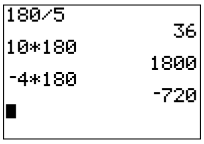
D2 $x_P = \cos 205^\circ \approx -0,91$ en $y_P = \sin 205^\circ \approx -0,42.$
$x_Q = \cos(-37^\circ) \approx 0,80$ en $y_Q = \sin(-37^\circ) \approx -0,60.$



D3a $\frac{1}{5}\pi$ rad $= \frac{1}{5} \cdot 180^\circ = 36^\circ.$

D3b 10π rad $= 10 \cdot 180^\circ = 1800^\circ.$

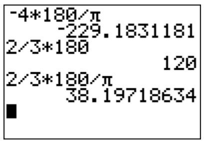
D3c -4π rad $= -4 \cdot 180^\circ = -720^\circ.$



D3d -4 rad $= -4 \cdot \frac{180^\circ}{\pi} \approx -229,2^\circ.$

D3e $\frac{2}{3}\pi$ rad $= \frac{2}{3} \cdot 180^\circ = 120^\circ.$

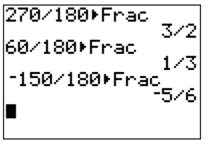
D3f $\frac{2}{3}$ rad $= \frac{2}{3} \cdot \frac{180^\circ}{\pi} \approx 38,2^\circ.$



D4a $270^\circ = \frac{270}{180} \cdot \pi$ rad $= 1\frac{1}{2}\pi$ rad.

D4b $60^\circ = \frac{60}{180} \cdot \pi$ rad $= \frac{1}{3}\pi$ rad.

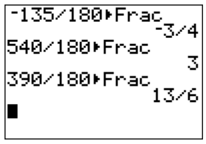
D4c $-150^\circ = \frac{-150}{180} \cdot \pi$ rad $= -\frac{5}{6}\pi$ rad.



D4d $-135^\circ = \frac{-135}{180} \cdot \pi$ rad $= -\frac{3}{4}\pi$ rad.

D4e $540^\circ = \frac{540}{180} \cdot \pi$ rad $= 3\pi$ rad.

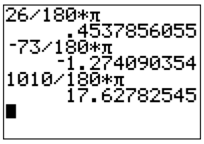
D4f $390^\circ = \frac{390}{180} \cdot \pi$ rad $= 2\frac{1}{6}\pi$ rad.



D5a $26^\circ = \frac{26}{180} \cdot \pi$ rad $\approx 0,45$ rad.

D5b $-73^\circ = \frac{-73}{180} \cdot \pi$ rad $\approx -1,27$ rad.

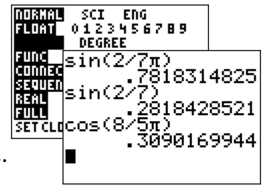
D5c $1010^\circ = \frac{1010}{180} \cdot \pi$ rad $\approx 17,63$ rad.



D6a $\sin(\frac{2}{7}\pi) \approx 0,78.$

D6b $\sin(\frac{2}{7}) \approx 0,28.$

D6c $\cos(1\frac{3}{5}\pi) \approx 0,31.$



D7a $\sin(\frac{5}{6}\pi) = \sin(\pi - \frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}.$

D7b $\cos(\frac{3}{4}\pi) = \cos(\pi - \frac{1}{4}\pi) = -\cos(\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}.$

D7c $\cos(1\frac{1}{3}\pi) = \cos(\pi + \frac{1}{3}\pi) = -\cos(\frac{1}{3}\pi) = -\frac{1}{2}.$

D8a $\sin(\alpha) = \frac{1}{2} = \sin(\frac{1}{6}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = \pi - \frac{1}{6}\pi = \frac{5}{6}\pi.$

D8b $\sin(\alpha) = -\frac{1}{2}\sqrt{2} = -\sin(\frac{1}{4}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi + \frac{1}{4}\pi = 1\frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$

D8c $\cos(\alpha) = \frac{1}{2}\sqrt{3} = \cos(\frac{1}{6}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = 2\pi - \frac{1}{6}\pi = 1\frac{5}{6}\pi.$

D9a
$\sin(2x + \frac{1}{2}\pi) = 0$
$2x + \frac{1}{2}\pi = k \cdot \pi$
$2x = -\frac{1}{2}\pi + k \cdot \pi$
$x = -\frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

D9b
$\cos(2x + \frac{1}{6}\pi) = 1$
$2x + \frac{1}{6}\pi = k \cdot 2\pi$
$2x = -\frac{1}{6}\pi + k \cdot 2\pi$
$x = -\frac{1}{12}\pi + k \cdot \pi.$

D9c
$\sin^2(\frac{1}{2}x) - \sin(\frac{1}{2}x) = 0$
$\sin(\frac{1}{2}x) \cdot (\sin(\frac{1}{2}x) - 1) = 0$
$\sin(\frac{1}{2}x) = 0$ of $\sin(\frac{1}{2}x) = 1$
$\frac{1}{2}x = k \cdot \pi$ of $\frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$
$x = k \cdot 2\pi$ of $x = \pi + k \cdot 4\pi.$

D10a
$\sin(\frac{1}{2}x + \pi) = \frac{1}{2}\sqrt{2}$
$\frac{1}{2}x + \pi = \frac{1}{4}\pi + k \cdot 2\pi$ of $\frac{1}{2}x + \pi = \pi - \frac{1}{4}\pi + k \cdot 2\pi$
$\frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $\frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi$
$x = -\frac{3}{2}\pi + k \cdot 4\pi$ of $x = -\frac{1}{2}\pi + k \cdot 4\pi.$

D10b
$\cos(-\frac{1}{3}x + \frac{1}{2}\pi) = -\frac{1}{2}.$
$-\frac{1}{3}x + \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi$ of $-\frac{1}{3}x + \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$
$-\frac{1}{3}x = \frac{1}{6}\pi + k \cdot 2\pi$ of $-\frac{1}{3}x = -\frac{7}{6}\pi + k \cdot 2\pi$
$x = -\frac{1}{2}\pi + k \cdot 6\pi$ of $x = \frac{7}{2}\pi + k \cdot 6\pi.$

D10c
$4\cos^2(\frac{1}{2}\pi x) = 3$
$\cos^2(\frac{1}{2}\pi x) = \frac{3}{4}$
$\cos(\frac{1}{2}\pi x) = \pm\sqrt{\frac{3}{4}} = \pm\sqrt{\frac{1}{4} \cdot 3} = \pm\frac{1}{2}\sqrt{3}$
$\frac{1}{2}\pi x = \pm\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \pm\frac{5}{6}\pi + k \cdot 2\pi$
$x = \pm\frac{1}{3} + k \cdot 4$ of $x = \pm\frac{5}{3} + k \cdot 4$
$x = -\frac{1}{3} + k \cdot 2$ of $x = \frac{1}{3} + k \cdot 2.$

D11a $2\sin(2x)=-\sqrt{3}$
$\sin(2x)=-\frac{1}{2}\sqrt{3}$
$2x=-\frac{1}{3}\pi+k\cdot 2\pi$ of $2x=\pi--\frac{1}{3}\pi+k\cdot 2\pi$
$x=-\frac{1}{6}\pi+k\cdot\pi$ of $2x=\frac{4}{3}\pi+k\cdot 2\pi.$
$x=-\frac{1}{6}\pi+k\cdot\pi$ of $x=\frac{2}{3}\pi+k\cdot\pi.$
x op $[0,\,2\pi]$ geeft
$x=\frac{5}{6}\pi$ of $x=1\frac{5}{6}\pi$ of $x=\frac{2}{3}\pi$ of $x=1\frac{2}{3}\pi.$

D11b $2\cos(1\frac{1}{2}x-\frac{1}{6}\pi)=-\sqrt{2}$
$\cos(1\frac{1}{2}x-\frac{1}{6}\pi)=-\frac{1}{2}\sqrt{2}$
$1\frac{1}{2}x-\frac{1}{6}\pi=\frac{3}{4}\pi+k\cdot 2\pi$ of $1\frac{1}{2}x-\frac{1}{6}\pi=-\frac{3}{4}\pi+k\cdot 2\pi$
$\frac{3}{2}x=\frac{11}{12}\pi+k\cdot 2\pi$ of $\frac{3}{2}x=-\frac{7}{12}\pi+k\cdot 2\pi$
$x=\frac{11}{18}\pi+k\cdot\frac{4}{3}\pi$ of $x=-\frac{7}{18}\pi+k\cdot\frac{4}{3}\pi.$
x op $[0,\,2\pi]$ geeft $x=\frac{11}{18}\pi$ of $x=\frac{35}{18}\pi$ of $x=\frac{17}{18}\pi.$

D11c $\sin^2(x)-\frac{1}{2}\sin(x)-\frac{1}{2}=0$
proberen met: $(\sin(x)-...)\cdot(\sin(x)+...)=0$ geeft
$(\sin(x)-1)\cdot(\sin(x)+\frac{1}{2})=0$
$\sin(x)=1$ of $\sin(x)=-\frac{1}{2}$
$x=\frac{1}{2}\pi+k\cdot 2\pi$ of $x=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x=\pi--\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{1}{2}\pi+k\cdot 2\pi$ of $x=-\frac{1}{6}\pi+k\cdot 2\pi$ of $x=1\frac{1}{6}\pi+k\cdot 2\pi.$
x op $[0,\,2\pi]$ geeft $x=\frac{1}{2}\pi$ of $x=1\frac{5}{6}\pi$ of $x=1\frac{1}{6}\pi.$

D12a $\sin(2x-1)=\sin(x+2)$
$2x-1=x+2+k\cdot 2\pi$ of $2x-1=\pi-(x+2)+k\cdot 2\pi$
$x=3+k\cdot 2\pi$ of $2x-1=\pi-x-2+k\cdot 2\pi$
$x=3+k\cdot 2\pi$ of $3x=\pi-1+k\cdot 2\pi$
$x=3+k\cdot 2\pi$ of $x=\frac{1}{3}\pi-\frac{1}{3}+k\cdot\frac{2}{3}\pi.$

D12b $\cos(x+\frac{1}{3}\pi)=\cos(2x-\frac{1}{2}\pi)$
$x+\frac{1}{3}\pi=2x-\frac{1}{2}\pi+k\cdot 2\pi$ of $x+\frac{1}{3}\pi=-(2x-\frac{1}{2}\pi)+k\cdot 2\pi$
$-x=-\frac{5}{6}\pi+k\cdot 2\pi$ of $x+\frac{1}{3}\pi=-2x+\frac{1}{2}\pi+k\cdot 2\pi$
$x=\frac{5}{6}\pi+k\cdot 2\pi$ of $3x=\frac{1}{6}\pi+k\cdot 2\pi$
$x=\frac{5}{6}\pi+k\cdot 2\pi$ of $x=\frac{1}{18}\pi+k\cdot\frac{2}{3}\pi.$

D12c $\sin(\frac{1}{2}\pi x)=\sin(\pi(x+1))$
$\frac{1}{2}\pi x=\pi(x+1)+k\cdot 2\pi$ of $\frac{1}{2}\pi x=\pi-\pi(x+1)+k\cdot 2\pi$
$\frac{1}{2}\pi x=\pi x+\pi+k\cdot 2\pi$ of $\frac{1}{2}\pi x=\pi-\pi x-\pi+k\cdot 2\pi$
$-\frac{1}{2}\pi x=\pi+k\cdot 2\pi$ of $1\frac{1}{2}\pi x=k\cdot 2\pi$
$x=-2+k\cdot 4$ of $x=k\cdot\frac{4}{3}.$

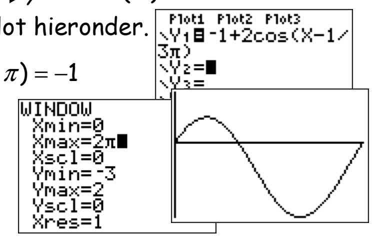
D13a $y=\sin(x)\xrightarrow{\text{translatie }(\frac{1}{2}\pi,0)}y=\sin(x-\frac{1}{2}\pi)\xrightarrow{\text{verm. }y\text{-as, }\frac{1}{3}}y=\sin(3x-\frac{1}{2}\pi)$ (moet in deze volgorde)
$y=\sin(3x-\frac{1}{2}\pi)\xrightarrow{\text{verm. }x\text{-as, }2}f(x)=2\sin(3x-\frac{1}{2}\pi).$ (deze vermenigvuldiging mag ook eerder staan)

D13b $y=\cos(x)\xrightarrow{\text{verm. }y\text{-as, }3}y=\cos(\frac{1}{3}x)\xrightarrow{\text{translatie }(-2,5)}g(x)=\cos(\frac{1}{3}(x+2))+5.$ (moet in deze volgorde)

D13c $y=\sin(x)\xrightarrow{\text{verm. }x\text{-as, }2}y=2\sin(x)\xrightarrow{\text{translatie }(\frac{1}{4}\pi,1)}y=2\sin(x-\frac{1}{4}\pi)+1$
$y=2\sin(x-\frac{1}{4}\pi)+1\xrightarrow{\text{verm. }y\text{-as, }\frac{1}{3}}h(x)=2\sin(3x-\frac{1}{4}\pi)+1$ (neem je beide translaties samen dan in deze volgorde)

D14 $y=\sin(x)\xrightarrow{\text{translatie }(\frac{1}{2}\pi,3)}y=\sin(x-\frac{1}{2}\pi)+3\xrightarrow{\text{verm. }y\text{-as, }\frac{1}{5}}f(x)=\sin(5x-\frac{1}{2}\pi)+3.$

D15a $y=\cos(x)\xrightarrow{\text{verm. }x\text{-as, }2}y=2\cos(x)\xrightarrow{\text{translatie }(\frac{1}{3}\pi,-1)}f(x)=2\cos(x-\frac{1}{3}\pi)-1.$ (in deze volgorde)
Maak een schets van de plot hieronder.



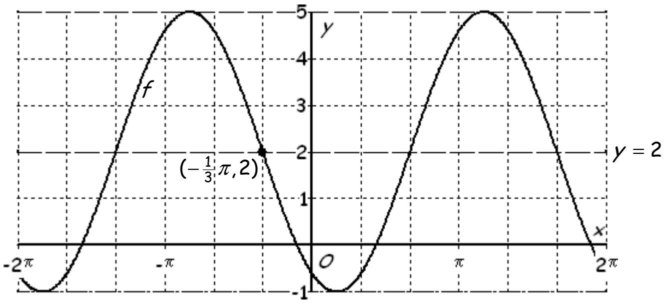
D15b $f(x)=y=-1+2\cos(x-\frac{1}{3}\pi)=-1$
$2\cos(x-\frac{1}{3}\pi)=0$
$\cos(x-\frac{1}{3}\pi)=0$
$x-\frac{1}{3}\pi=\frac{1}{2}\pi+k\cdot\pi$
$x=\frac{5}{6}\pi+k\cdot\pi$
x op $[0,\,2\pi]$ geeft $x=\frac{5}{6}\pi$ of $x=1\frac{5}{6}\pi.$

D15c $y=2\cos x$ heeft toppen $(0,\,2)$, $(\pi,\,-2)$ en $(2\pi,\,2)$.
Dus de toppen van f zijn: $(\frac{1}{3}\pi,\,1)$ en $(1\frac{1}{3}\pi,\,-3)$.

D15d $f(x)=y=-1+2\cos(x-\frac{1}{3}\pi)=0$
$2\cos(x-\frac{1}{3}\pi)=1$
$\cos(x-\frac{1}{3}\pi)=\frac{1}{2}$
$x-\frac{1}{3}\pi=\frac{1}{3}\pi+k\cdot 2\pi$ of $x-\frac{1}{3}\pi=-\frac{1}{3}\pi+k\cdot 2\pi$
$x=\frac{2}{3}\pi+k\cdot 2\pi$ of $x=k\cdot 2\pi$
x op $[0,\,2\pi]$ geeft $x=\frac{2}{3}\pi$ of $x=0$ of $x=2\pi.$

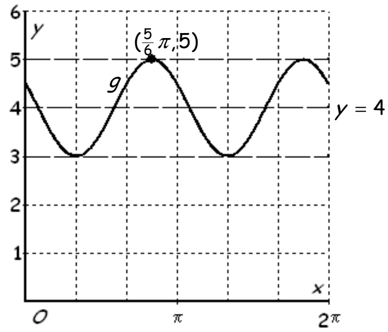
D16a $f(x) = 2 - 3\sin(x + \frac{1}{3}\pi)$.

$\begin{cases} \text{evenwichtsstand } 2 \\ \text{amplitude } 3 \\ \text{periode } \frac{2\pi}{1} = 2\pi \\ -3 < 0 \Rightarrow \text{dalend door evenwichtsstand in } (-\frac{1}{3}\pi, 2) \end{cases}$



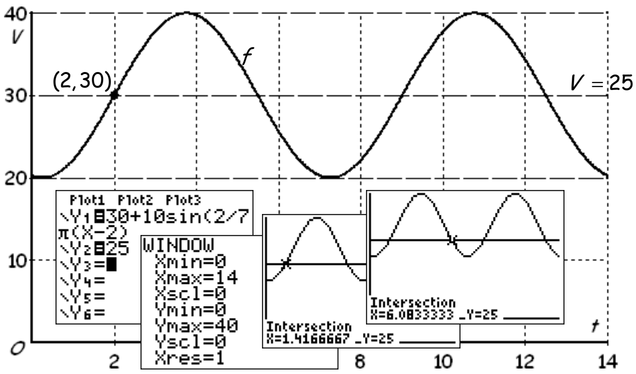
D16b $g(x) = 4 + \cos(2x - 1\frac{2}{3}\pi) = 4 + \cos(2(x - \frac{5}{6}\pi))$

$\begin{cases} \text{evenwichtsstand } 4 \\ \text{amplitude } 1 \\ \text{periode } \frac{2\pi}{2} = \pi \\ 1 > 0 \Rightarrow \text{beginpunt } (\frac{5}{6}\pi, 5) \text{ is hoogste punt} \end{cases}$



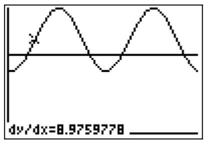
D17a $f(x) = 30 + 10\sin(\frac{2}{7}\pi(t - 2))$.

$\begin{cases} \text{evenwichtsstand } 30 \\ \text{amplitude } 10 \\ \text{periode } \frac{2\pi}{\frac{2}{7}\pi} = 7 \\ 10 > 0 \Rightarrow \text{stijgend door evenwichtsstand in } (2, 30) \end{cases}$



D17b $V = 25$ (intersect) $\Rightarrow$ (bedenk dat de periode 7)
$t \approx 1{,}42$ of $t \approx 6{,}08$ of $t \approx 8{,}42$ of $t \approx 13{,}08$.
Met gebruik van de plot (of de grafiek) vind je:
$V > 25$ op $[0, 14]$ voor $1{,}42 < t < 6{,}08$ of $8{,}42 < t < 13{,}08$.

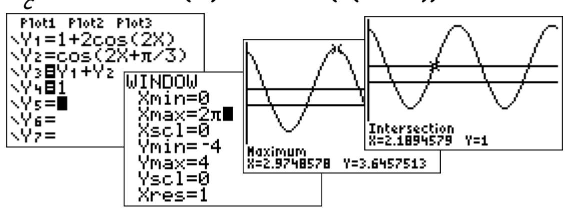
D17c De grootste helling is $\left[\frac{dV}{dt}\right]_{t=2} \approx 8{,}98$.
(maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat)



D18a $N = a + b\sin(c(t - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{65 + -35}{2} = \frac{30}{2} = 15$; b (= –amplitude) $= -(65 - 15) = -50$;
c (= $\frac{2\pi}{\text{periode}}$) $= \frac{2\pi}{30} = \frac{1}{15}\pi$ en $d = 25$ (sinus gaat dalend door de evenwichtsstand voor $t = 25$) $\Rightarrow N = 15 - 50\sin(\frac{1}{15}\pi(t - 25))$.

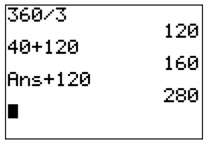
D18b $N = a + b\cos(c(t - d))$ met $a = 15$; b (= amplitude) $= 50$;
$c = \frac{1}{15}\pi$ en $d = 17{,}5$ (cosinus heeft maximum voor $t = \frac{10 + 25}{2} = \frac{35}{2} = 17{,}5$) $\Rightarrow N = 15 + 50\sin(\frac{1}{15}\pi(t - 17{,}5))$.

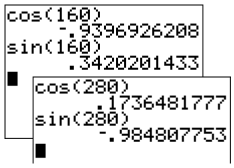
D19 De evenwichtsstand van f is 1 en die van g is 0 $\Rightarrow$ evenwichtsstand van h is $1 + 0 = 1 \Rightarrow a = 1$.
De periode van f en van g zijn beide $\pi \Rightarrow$ periode van h is $\pi = \frac{2\pi}{c} \Rightarrow c = 2 \Rightarrow h(x) = 1 + b\sin(2(x - d))$.
$h(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).
Optie maximum geeft maximum $h(2{,}97) \approx 3{,}65 \Rightarrow b \approx 2{,}65$.
(b = amplitude = maximum – evenwichtsstand = $3{,}65 - 1 = 2{,}65$)
$h(x) = 1$ (intersect) $\Rightarrow x \approx 2{,}19 = d$.
(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand)
Dus $h(x) = 1 + 2{,}65\sin(2(x - 2{,}19))$.



Gemengde opgaven 6. Goniometrische formules

G13 $\angle AOB = \angle BOC = \angle COA = 120°$ (alle drie even groot en samen een volle hoek).
$x_A = \cos 40° \approx 0{,}766$ en $y_A = \sin 40° \approx 0{,}643$.
$x_B = \cos 160° \approx -0{,}940$ en $y_B = \sin 160° \approx 0{,}342$.
$x_C = \cos 280° \approx 0{,}174$ en $y_C = \sin 280° \approx -0{,}985$.



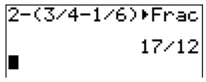


G14a $x_A = \cos(\alpha) = \cos\left(\frac{2}{3}\pi\right) = \cos\left(\pi - \frac{1}{3}\pi\right) = -\cos\left(\frac{1}{3}\pi\right) = -\frac{1}{2}$ en $y_A = \sin(\alpha) = \sin\left(\frac{2}{3}\pi\right) = \sin\left(\pi - \frac{1}{3}\pi\right) = \sin\left(\frac{1}{3}\pi\right) = \frac{1}{2}\sqrt{3}$.

G14b $x_C = \cos(\gamma) = \cos\left(-\frac{1}{6}\pi\right) = \cos\left(\frac{1}{6}\pi\right) = \frac{1}{2}\sqrt{3}$ en $y_C = \sin(\gamma) = \sin\left(-\frac{1}{6}\pi\right) = -\sin\left(\frac{1}{6}\pi\right) = -\frac{1}{2}$.

G14c $x_B = \cos(\beta) = -\frac{1}{2}\sqrt{2} = -\cos\left(\frac{1}{4}\pi\right)$ (en ligt in het 3^e kwart = kwadrant III) $\Rightarrow \beta = -\pi + \frac{1}{4}\pi = -\frac{3}{4}\pi$.

G14d De langste cirkelboog BC is de boog van C via A naar B.
De lengte van deze boog = de omtrek van de cirkel − de lengte van de kortste boog BC
$= 2\pi - (\frac{3}{4}\pi - \frac{1}{6}\pi) = 2\pi - (\frac{9}{12}\pi - \frac{2}{12}\pi) = 2\pi - \frac{7}{12}\pi = 1\frac{5}{12}\pi$.



G15a In 12 seconden één rondgang ⇒ in 12 seconden een hoek van 2π rad.
In 2 seconden een hoek van $\frac{2}{12} \cdot 2\pi$ rad $= \frac{1}{3}\pi$ rad $\Rightarrow x_P = \cos(\frac{1}{3}\pi) = \frac{1}{2}$ en $y_P = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$.
In 7,5 seconden een hoek van $\frac{7{,}5}{12} \cdot 2\pi$ rad $= 1\frac{1}{4}\pi$ rad $\Rightarrow x_P = \cos(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$ en $y_P = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$.
In 11 seconden een hoek van $\frac{11}{12} \cdot 2\pi$ rad $= \frac{11}{6}\pi$ rad $\Rightarrow x_P = \cos(\frac{11}{6}\pi) = \cos(-\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$ en $y_P = \sin(-\frac{1}{6}\pi) = -\frac{1}{2}$.

G15b $x_P = -\frac{1}{2} = \cos(\frac{2}{3}\pi) \Rightarrow$ draaiïngshoek $\frac{2}{3}\pi$ (in kwadrant II) of $2\pi - \frac{2}{3}\pi = \frac{4}{3}\pi$ (in kwadrant III).
Dit is na $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ rondgang of na $\frac{\frac{4}{3}\pi}{2\pi} = \frac{2}{3}$ rondgang. Dus na $\frac{1}{3} \cdot 12 = 4$ seconden of na $\frac{2}{3} \cdot 12 = 8$ seconden.

G16a $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$
$3x - \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi$ of $3x - \frac{1}{2}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$
$3x = \frac{3}{4}\pi + k \cdot 2\pi$ of $3x = \frac{1}{4}\pi + k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$.

G16b $\sin(\frac{1}{3}x + \frac{1}{4}\pi) = -\frac{1}{2}$
$\frac{1}{3}x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{3}x + \frac{1}{4}\pi = \pi - -\frac{1}{6}\pi + k \cdot 2\pi$
$\frac{1}{3}x = -\frac{5}{12}\pi + k \cdot 2\pi$ of $\frac{1}{3}x = \frac{11}{12}\pi + k \cdot 2\pi$
$x = -\frac{5}{4}\pi + k \cdot 6\pi$ of $x = \frac{11}{4}\pi + k \cdot 6\pi$.

G16c $\sin(\frac{1}{2}x - \frac{1}{3}\pi) \cdot \cos(2x) = 0$
$\sin(\frac{1}{2}x - \frac{1}{3}\pi) = 0$ of $\cos(2x) = 0$
$\frac{1}{2}x - \frac{1}{3}\pi = k \cdot \pi$ of $2x = \frac{1}{2}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{1}{3}\pi + k \cdot \pi$ of $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
$x = \frac{2}{3}\pi + k \cdot 2\pi$ of $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

G16d $4\cos^2(2\pi x - \frac{1}{2}\pi) = 3$
$\cos^2(2\pi x - \frac{1}{2}\pi) = \frac{3}{4}$
$\cos(2\pi x - \frac{1}{2}\pi) = \pm\sqrt{\frac{3}{4}} = \pm\sqrt{\frac{1}{4} \cdot 3} = \pm\frac{1}{2}\sqrt{3}$
$2\pi x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $2\pi x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
of $2\pi x - \frac{1}{2}\pi = \frac{5}{6}\pi + k \cdot 2\pi$ of $2\pi x - \frac{1}{2}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
$2\pi x = \frac{2}{3}\pi + k \cdot 2\pi$ of $2\pi x = \frac{1}{3}\pi + k \cdot 2\pi$
of $2\pi x = \frac{4}{3}\pi + k \cdot 2\pi$ of $2\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{1}{3} + k \cdot 1$ of $x = \frac{1}{6} + k \cdot 1$ of $x = \frac{2}{3} + k \cdot 1$ of $x = -\frac{1}{6} + k \cdot 1$
$x = \frac{1}{3} + k \cdot \frac{1}{2}$ of $x = \frac{1}{6} + k \cdot \frac{1}{2}$.

G17a $\cos(2x - \frac{1}{2}\pi) = \cos(\pi - x)$
$2x - \frac{1}{2}\pi = \pi - x + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = -(\pi - x) + k \cdot 2\pi$
$3x = 1\frac{1}{2}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = -\pi + x + k \cdot 2\pi$
$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$ of $x = -\frac{1}{2}\pi + k \cdot 2\pi$.

G17b $\sin(2x + \frac{1}{3}\pi) = \sin(x - \frac{1}{2}\pi)$
$2x + \frac{1}{3}\pi = x - \frac{1}{2}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$
$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$
$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $3x = \frac{7}{6}\pi + k \cdot 2\pi$
$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$.

G17c $\sin(\pi x) = \sin(2\pi x)$
$\pi x = 2\pi x + k \cdot 2\pi$ of $\pi x = \pi - 2\pi x + k \cdot 2\pi$
$-\pi x = k \cdot 2\pi$ of $3\pi x = \pi + k \cdot 2\pi$
$x = k \cdot 2$ of $x = \frac{1}{3} + k \cdot \frac{2}{3}$.

G17d $\cos(10\pi x) = \cos(5\pi x - 6\pi)$
$10\pi x = 5\pi x - 6\pi + k \cdot 2\pi$ of $10\pi x = -(5\pi x - 6\pi) + k \cdot 2\pi$
$5\pi x = -6\pi + k \cdot 2\pi$ of $10\pi x = -5\pi x + 6\pi + k \cdot 2\pi$
$x = -\frac{6}{5} + k \cdot \frac{2}{5}$ of $15\pi x = 6\pi + k \cdot 2\pi$
$x = -\frac{6}{5} + k \cdot \frac{2}{5}$ of $x = \frac{2}{5} + k \cdot \frac{2}{15}$
$x = k \cdot \frac{2}{5}$ of $x = k \cdot \frac{2}{15}$
$x = k \cdot \frac{2}{15}$.

G18a $\sin(1\frac{1}{2}x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$
$1\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot 2\pi$ of $1\frac{1}{2}x - \frac{1}{6}\pi = \pi - \frac{1}{3}\pi + k \cdot 2\pi$
$\frac{3}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$ of $\frac{3}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{3}\pi + k \cdot \frac{4}{3}\pi$ of $x = \frac{5}{9}\pi + k \cdot \frac{4}{3}\pi$.
x op $[0,\ 2\pi]$ geeft
$x = \frac{1}{3}\pi$ of $x = \frac{5}{3}\pi$ of $x = \frac{5}{9}\pi$ of $x = \frac{17}{9}\pi$.

G18b $\cos^3(2\frac{1}{2}x) + \cos(2\frac{1}{2}x) = 0$
$\cos(2\frac{1}{2}x) \cdot (\cos^2(2\frac{1}{2}x) + 1) = 0$
$\cos(2\frac{1}{2}x) = 0$ of $\cos^2(2\frac{1}{2}x) = -1$ (kan niet)
$\frac{5}{2}x = \frac{1}{2}\pi + k \cdot \pi$
$x = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$.
x op $[0,\ 2\pi]$ geeft:
$x = \frac{1}{5}\pi$ of $x = \frac{3}{5}\pi$ of $x = \pi$ of $x = \frac{7}{5}\pi$ of $x = \frac{9}{5}\pi$.

G18c $\sin^2(1{,}5x) = \sin(1{,}5x) + 2$
$\sin^2(1{,}5x) - \sin(1{,}5x) - 2 = 0$
proberen $(\sin(1{,}5x) - ...) \cdot (\sin(1{,}5x) + ...) = 0$ geeft
$(\sin(1{,}5x) - 2) \cdot (\sin(1{,}5x) + 1) = 0$
$\sin(1{,}5x) = 1$ (kan niet) of $\sin(1{,}5x) = -1$
$\frac{3}{2}x = -\frac{1}{2}\pi + k \cdot 2\pi$
$x = -\frac{1}{3}\pi + k \cdot \frac{4}{3}\pi$.
x op $[0,\ 2\pi]$ geeft $x = \pi$.

G18d $\cos(2x + \frac{1}{3}\pi) = \cos(3x - \frac{1}{6}\pi)$
$2x + \frac{1}{3}\pi = 3x - \frac{1}{6}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = -(3x - \frac{1}{6}\pi) + k \cdot 2\pi$
$-x = -\frac{1}{2}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = -3x + \frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{2}\pi + k \cdot 2\pi$ of $5x = -\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{2}\pi + k \cdot 2\pi$ of $x = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$.
x op $[0,\ 2\pi]$ geeft: $x = \frac{1}{2}\pi$ of $x = \frac{11}{30}\pi$ of $x = \frac{23}{30}\pi$ of
$x = \frac{35}{30}\pi$ of $x = \frac{47}{30}\pi$ of $x = \frac{59}{30}\pi$.

G19a $y = \cos(x) \xrightarrow{\text{translatie } (0,-2)} y = \cos(x) - 2 \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3 \cdot (\cos(x) - 2) = 3\cos(x) - 6$.

G19b $y = \cos(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} y = \cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = \cos(2x - \frac{1}{3}\pi)$.

G19c $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{translatie } (0,-2)} y = 3\cos(x) - 2$
$y = 3\cos(x) - 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x) - 2$.

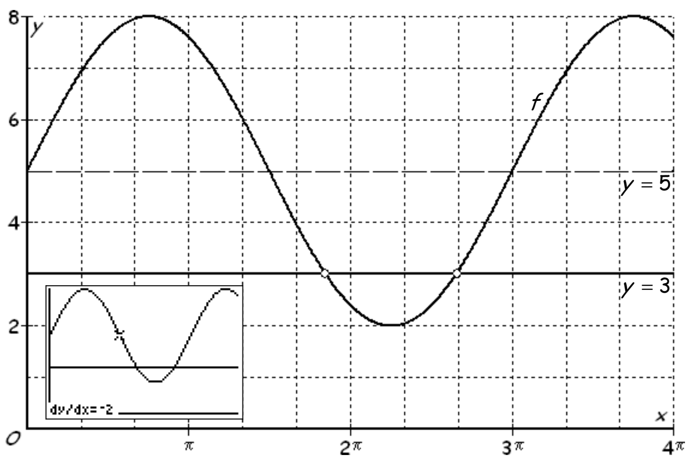
G19d $y = \cos(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = \cos(2x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(2x)$
$y = 3\cos(2x) \xrightarrow{\text{translatie } (0,-2)} y = 3\cos(2x) - 2$.

G19e $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x)$
$y = 3\cos(2x) \xrightarrow{\text{translatie } (0,-2)} y = 3\cos(2x) - 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2 \cdot 2x) - 2 = 3\cos(4x) - 2$.

G19f $y = \cos(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} y = \cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x - \frac{1}{3}\pi)$
$y = 3\cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x - \frac{1}{3}\pi) \xrightarrow{\text{translatie } (0,-2)} y = 3\cos(2x - \frac{1}{3}\pi) - 2$.

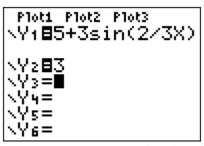
G20a $y = \sin(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{3}{2}]{\text{verm. } x\text{-as, } 3} y = 3\sin(\frac{2}{3}x) \xrightarrow{\text{translatie } (0,5)} f(x) = 3\sin(\frac{2}{3}x) + 5$.

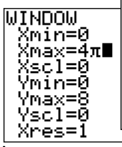
G20b Evenwichtsstand 5, amplitude 3, periode $\frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$ en $3 > 0$ dus grafiek stijgend door beginpunt (0, 5).
Zie de grafiek hiernaast.

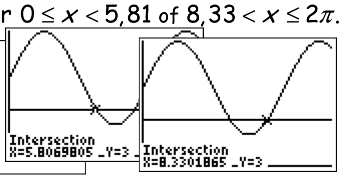


G20c $B_f = [3 \cdot -1 + 5,\ 3 \cdot 1 + 5] = [2,\ 8]$.

G20d $5 + 3\sin(\frac{2}{3}x) = 3$ (intersect) $\Rightarrow x \approx 5{,}81$ of $x \approx 8{,}33$.
Met behulp van de grafiek lees je dan af:
$f(x) > 3$ op $[0, 2\pi]$ voor $0 \le x < 5{,}81$ of $8{,}33 < x \le 2\pi$.

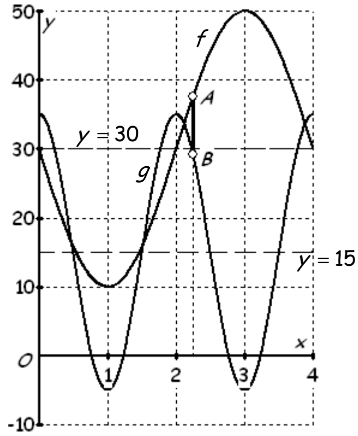






G20e Minimale helling (grootste daling) in een punt van f door de evenwichtsstand $\Rightarrow$ in $(1\frac{1}{2}\pi,\ 5)$.
$\left[\frac{dy}{dx}\right]_{x=1\frac{1}{2}\pi} = -2 \Rightarrow$ de kleinste helling is -2.

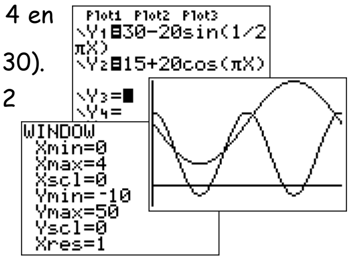
G21a $y = \sin(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{2}{\pi}]{\text{verm. } x\text{-as, } -20} y = -20\sin(\frac{\pi}{2}x) \xrightarrow{\text{translatie } (0,30)} f(x) = -20\sin(\frac{\pi}{2}x) + 30$.
$y = \cos(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{1}{\pi}]{\text{verm. } x\text{-as, } 20} y = 20\cos(\pi x) \xrightarrow{\text{translatie } (0,15)} g(x) = 20\cos(\pi x) + 15$.



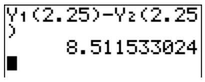
G21b 🖩 f: evenwichtsstand 30, amplitude 20, periode $\frac{2\pi}{\frac{1}{2}\pi} = 4$ en $-20 < 0$ dus dalend door evenwichtsstand in (0, 30).

g: evenwichtsstand 15, amplitude 20, periode $\frac{2\pi}{\pi} = 2$ en $20 > 0$ dus beginpunt (0, 35) is hoogste punt.

Zie de grafieken hiernaast.



G21c 🖩 $AB = f(2{,}25) - g(2{,}25) \approx 8{,}51$.

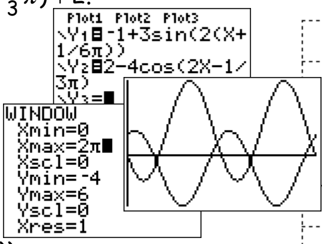


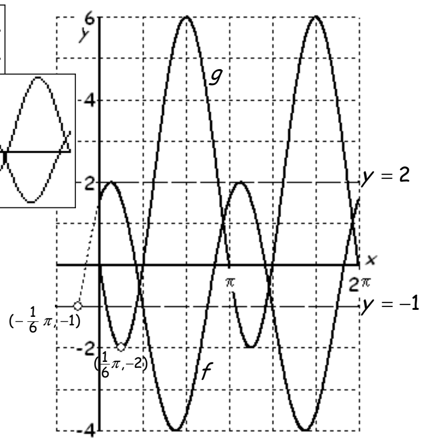
G21d 🖩 Voor $-5 \le p < 10$ heeft $g(x) = p$ wel oplossingen en $f(x) = p$ geen oplossingen.

G22a 🖩 $y = \sin(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{1}{2}]{\text{verm. } x\text{-as, } 3} y = 3\sin(2x) \xrightarrow{\text{translatie } (-\frac{1}{6}\pi, -1)} f(x) = 3\sin(2(x + \frac{1}{6}\pi)) - 1.$

$y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } -4} y = -4\cos(\pi x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 2)} y = -4\cos(x - \frac{1}{3}\pi) + 2$

$y = -4\cos(x - \frac{1}{3}\pi) + 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} g(x) = -4\cos(2x - \frac{1}{3}\pi) + 2.$





G22b 🖩 f: evenwichtsstand -1, amplitude 3, periode $\frac{2\pi}{2} = \pi$ en $3 > 0$ dus stijgend door evenwichtsstand in $(-\frac{1}{6}\pi,\ -1)$.

g: evenwichtsstand 2, amplitude 4, periode $\frac{2\pi}{2} = \pi$ en $-4 < 0$ dus beginpunt $(\frac{1}{6}\pi,\ -2)$ is laagste punt.

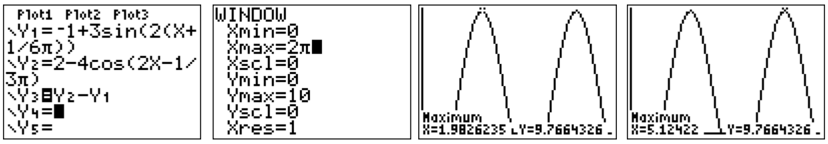
Zie de grafieken hiernaast.

G22c 🖩 $AB = g(a) - f(a) = 2 - 4\cos(2a - \frac{1}{3}\pi) - (-1 + 3\sin(2(a + \frac{1}{6}\pi)))$. (met $AB > 0$)

Voer deze formule in op de GR (schakel de andere formules uit door ENTER op =).

Optie maximum geeft $x \approx 1{,}98$ met $y \approx 9{,}77$ en $x \approx 5{,}12$ met $y \approx 9{,}77$.

Dus AB is maximaal (9,77) voor $a \approx 1{,}98$ of $a \approx 5{,}12$.



G22d 🖩 Voer de formule $h(x) = f(x) + g(x)$ in op de GR.

Optie maximum geeft $x \approx 2{,}504$ met $y \approx 3{,}053$.

Optie minimum geeft $x \approx 0{,}933$ met $y \approx -1{,}053$.

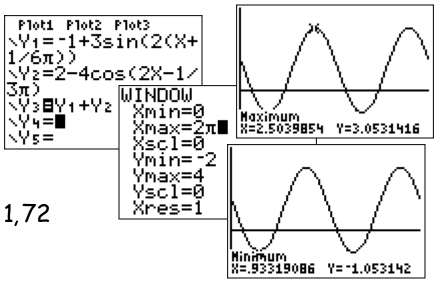
De evenwichtsstand is $-1 + 2 = 1$ (of $\frac{3{,}053 + -1{,}053}{2}$).

De grafiek gaat stijgend door de evenwichtsstand voor $x \approx \frac{2{,}504 + 0{,}933}{2} \approx 1{,}72$

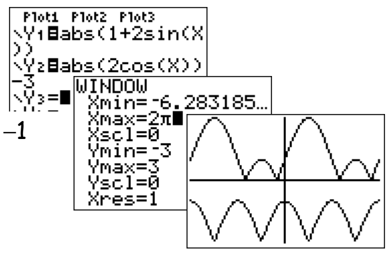
De amplitude is $\frac{3{,}052 - -1{,}053}{2} \approx 2{,}05$.

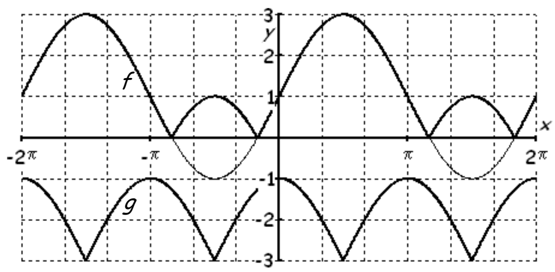
De periode is π (f en g hebben beide periode π) $\Rightarrow c = 2$.

Dus $h(x) = 1 + 2{,}05\sin(2(x - 1{,}72))$.



G23a 🖩 Zie de grafieken hiernaast.





G23b 🖩 $|1 + 2\sin(x)| = 1$

$1 + 2\sin(x) = 1$ of $1 + 2\sin(x) = -1$

$2\sin(x) = 0$ of $2\sin(x) = -2$

$\sin(x) = 0$ of $\sin(x) = -1$

$x = k \cdot \pi$ of $x = -\frac{1}{2}\pi + k \cdot 2\pi$

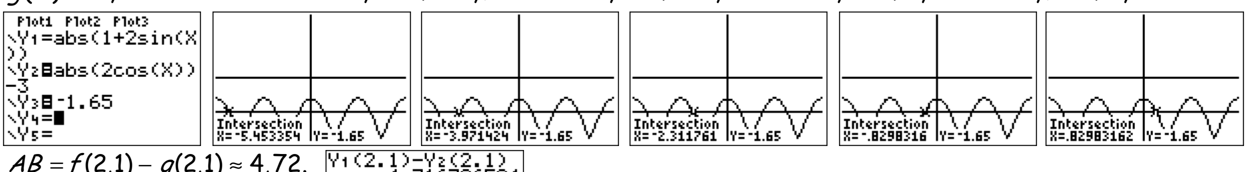
x op $[-2\pi,\ 2\pi]$ geeft: $x = -2\pi$ of $x = -\pi$ of $x = 0$ of $x = \pi$ of $x = 2\pi$ of $x = -\frac{1}{2}\pi$ of $x = 1\frac{1}{2}\pi$.

G23c 🖩 $g(x) = -1{,}65 \Rightarrow |2\cos(x)| - 3 = -1{,}65$ (intersect) $\Rightarrow$

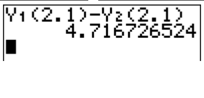
$x \approx -5{,}45$ of $x \approx -3{,}97$ of $x \approx -2{,}31$ of $x \approx -0{,}83$ of $x \approx 0{,}83$ of $x \approx 2{,}31$ of $x \approx 3{,}97$ of $x \approx 5{,}45$.

Nu aflezen in de plot (of de grafiek):

$g(x) > -1{,}65$ voor $-2\pi \le x < -5{,}45$ of $-3{,}97 < x < -2{,}31$ of $-0{,}83 < x < 0{,}83$ of $2{,}31 < x < 3{,}97$ of $5{,}45 < x \le 2\pi$.



G23d 🖩 $AB = f(2{,}1) - g(2{,}1) \approx 4{,}72$.



G24a In de tweede figuur is dat $6\,500 - 1500 = 5\,000$ (cm^3).

G24b Lees in de tweede figuur af: $V = 5\,500$ voor $t = 1\frac{1}{2}$ of $t = 6$ (sec.).
Dus per periode (van 15 sec.) is $6 - 1\frac{1}{2} = 4\frac{1}{2}$ seconde meer dan $5\,500$ cm^3 lucht in de longen.
Dus per minuuut (= 60 sec.) is $4 \cdot 4\frac{1}{2} = 18$ seconden meer dan $5\,500$ cm^3 lucht in de longen.

G24c In de eerste figuur is de periode 6 seconden, dus er zijn 10 ademhalingen per minuut.
Het *minuutvolume* is $10 \cdot 500 = 5\,000$ (cm^3, dus 5 liter).
In de tweede figuur is de periode 15 seconden, dus er zijn 4 ademhalingen per minuut.
Het *minuutvolume* is $4 \cdot 5\,000 = 20\,000$ (cm^3, dus 20 liter).
De verhouding van het *minuutvolume* bij de ademritmes van de eerste en tweede figuur is $5\,000 : 20\,000 = 1 : 4$.

G24d $V = a + b\sin(c(t - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{4000 + 3500}{2} = 3\,750$; b (= amplitude) = $4\,000 - 3\,750 = 250$;
c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{6} = \frac{1}{3}\pi$ en $d = 0$ (sinus gaat stijgend door de evenwichtsstand voor $t = 0$) $\Rightarrow V = 3\,750 + 250\sin(\frac{1}{3}\pi t)$.

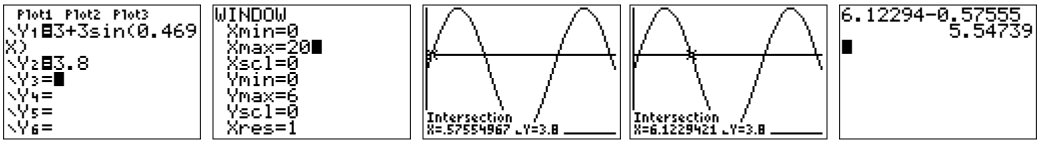
G24e $V = a + b\cos(c(t - d))$ met a (= evenwichtsstand) = $\frac{6500 + 1500}{2} = 4\,000$; b (= amplitude) = $6\,500 - 4\,000 = 2\,500$;
c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{15} = \frac{2}{15}\pi$ en $d = \frac{15}{4}$ (cosinus heeft maximum voor $t = \frac{15}{4}$) $\Rightarrow V = 4\,000 + 2\,500\cos(\frac{2}{15}\pi(t - \frac{15}{4}))$.

G24f $V = a - b\cos(c(t - d))$ met a (= evenwichtsstand) = $4\,200$ (gegeven); b (= −amplitude) = $-2\,500$;
(de periode van één ademhaling is $1\frac{1}{2}$ seconde, want er zijn 40 ademhalingen per minuut $\Rightarrow$)
c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{1,5} = \frac{20}{15}\pi = \frac{4}{3}\pi$ en $d = 0$ (gegeven: cosinus heeft minimum voor $t = 0$) $\Rightarrow V = 4\,200 - 2\,500\cos(\frac{4}{3}\pi t)$.

G25a $f(x) = a + b\sin(c(x - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{3\frac{1}{2} + -1\frac{1}{2}}{2} = \frac{2}{2} = 1$; b (= amplitude) = $3\frac{1}{2} - 1 = 2\frac{1}{2}$;
c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{\frac{4}{3}\pi} = 2 \cdot \frac{3}{4} = 1\frac{1}{2}$ en $d = \frac{1}{3}\pi$ (sinus stijgend door evenwichtsstand voor $x = \frac{1}{3}\pi$) $\Rightarrow f(x) = 1 + 2\frac{1}{2}\sin(1\frac{1}{2}(x - \frac{1}{3}\pi))$.

G25b $N = a + b\cos(c(t - d))$ met a (= evenwichtsstand) = $\frac{40 + 0}{2} = 20$; b (= amplitude) = $40 - 20 = 20$;
(tussen het maximum en het minimum ligt een halve periode $\Rightarrow$ de periode is $2 \times (7 - 1) = 2 \times 6 = 12 \Rightarrow$)
c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{12} = \frac{1}{6}\pi$ en $d = 1$ (cosinus heeft maximum voor $t = 1$) $\Rightarrow N = 20 + 20\cos(\frac{1}{6}\pi(t - 1))$.

G26a $y = 3 + 3\sin(0,469x) = 3,8$ (met intersect twee snijpunten naast één top) $\Rightarrow x \approx 0,57555$ of $x \approx 6,12294$.
Dus de breedte van het blokje is $6,12294 - 0,57555 \approx 5,5$ (cm).



G26b $SQ = \sqrt{55^2 + 67^2} \approx 86,68$.
S ligt even hoog als P; hetzelfde aantal golven; even hoge toppen; enz.
De nieuwe grafiek is een horizontale uitrekking van G.11 met factor $\frac{86,68}{67}$.
$y = 3 + 3\sin(0,469x) \xrightarrow{\text{verm. } y\text{-as, } \frac{86,68}{67}} y = 3 + 3\sin(0,469 \cdot \frac{67}{86,68}x)$
Dus $y = 3 + 3\sin(0,363x)$.